# In Transit to Constant Time Shortest-Path Queries in Road Networks

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- for general graphs with non-negative edge weights, exact solution given by Dijkstra's algorithm in $O(m + n \log n)$ where $n = \#$ nodes, $m = \#$ edges

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- $\Rightarrow$ need to exploit the special structure of roadmaps
- so far, best solutions allow for a query time in the order of *milliseconds* (with preprocessing)

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- yields shortest path queries in the order of *milliseconds* on the US roadmap (after preprocessing)
- any sensible reason to aim for faster query times ?
  $\Rightarrow$YES! Web services, traffic simulations, logistics ...

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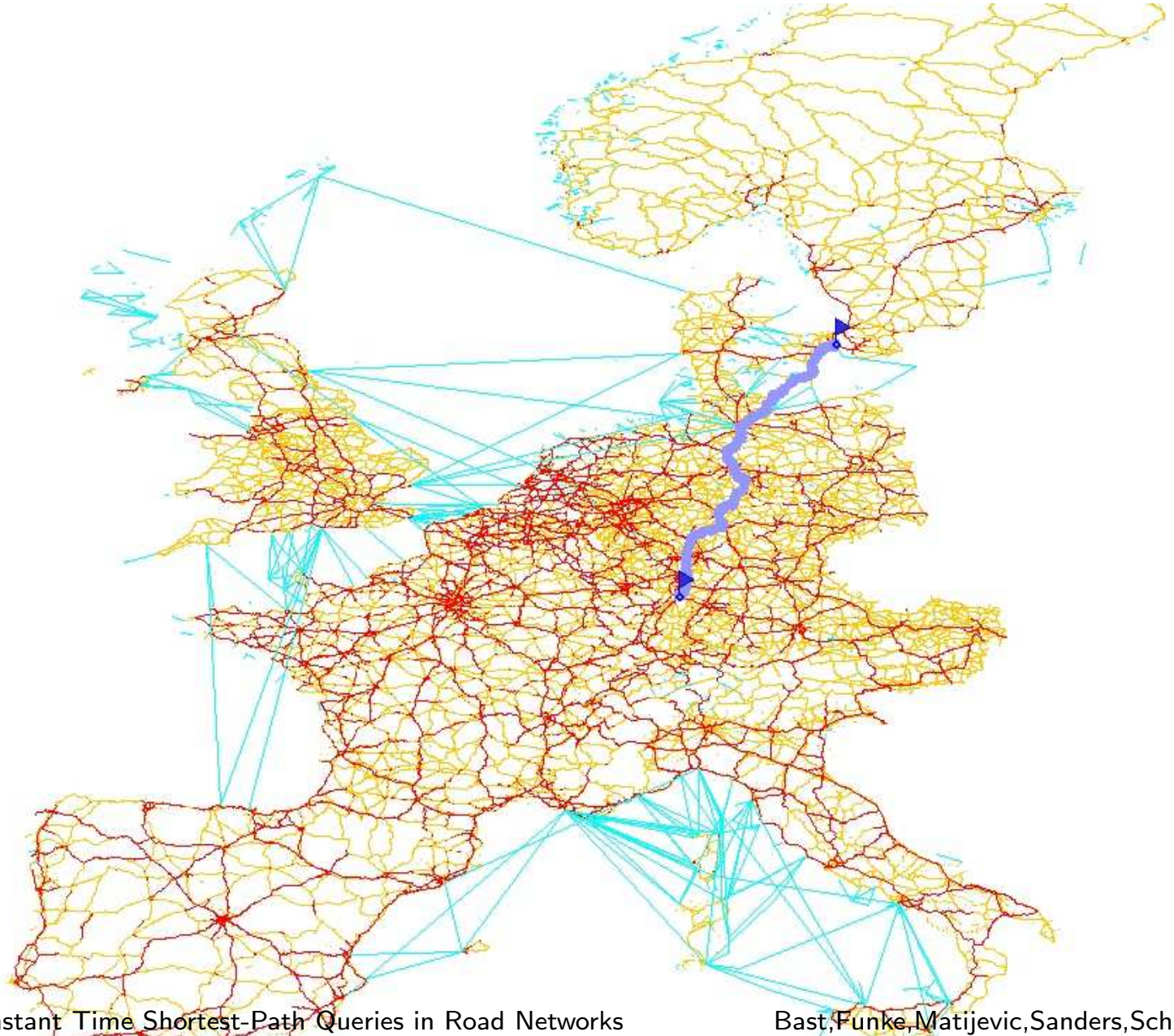
# Example: Karlsruhe → Copenhagen



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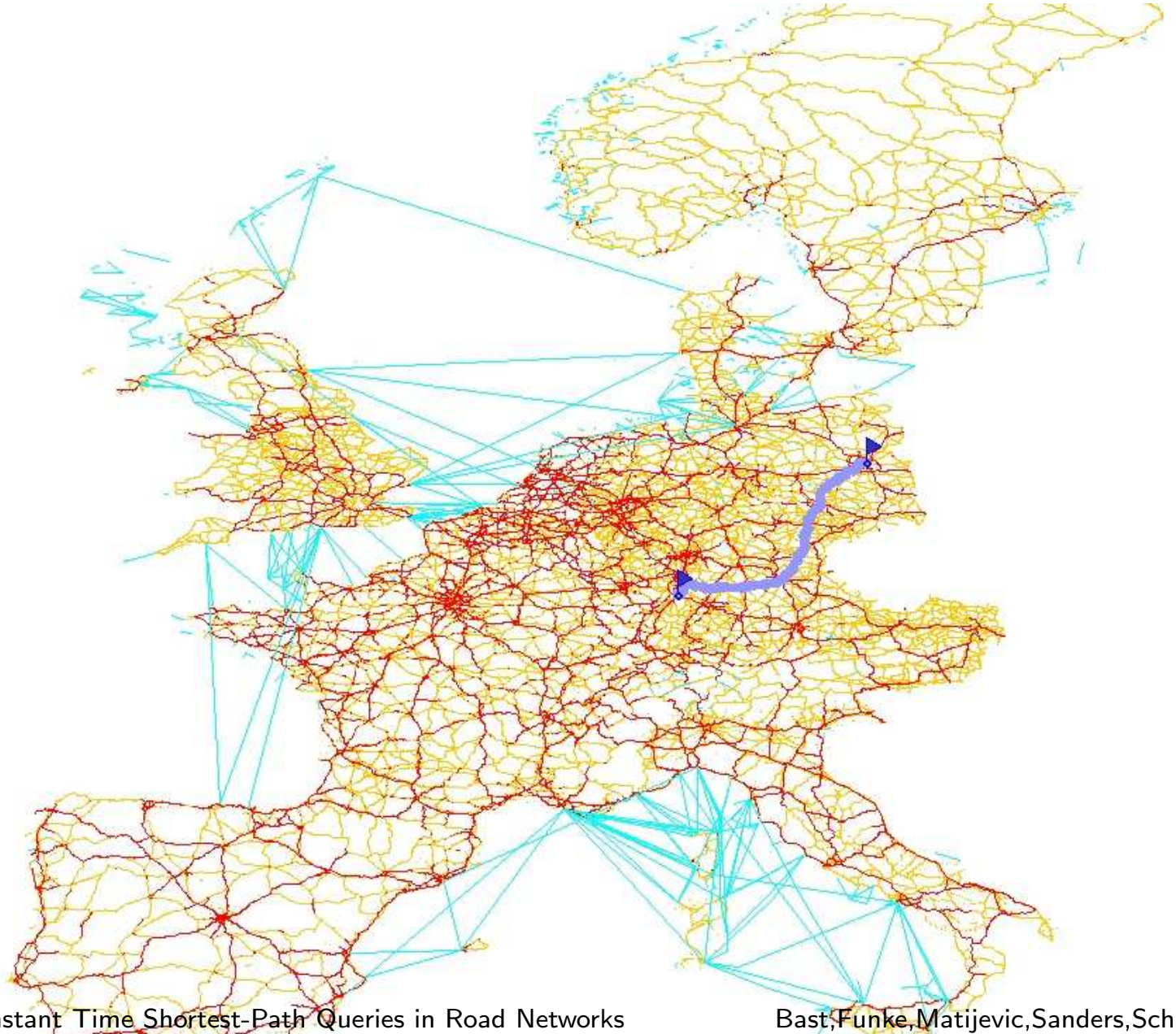

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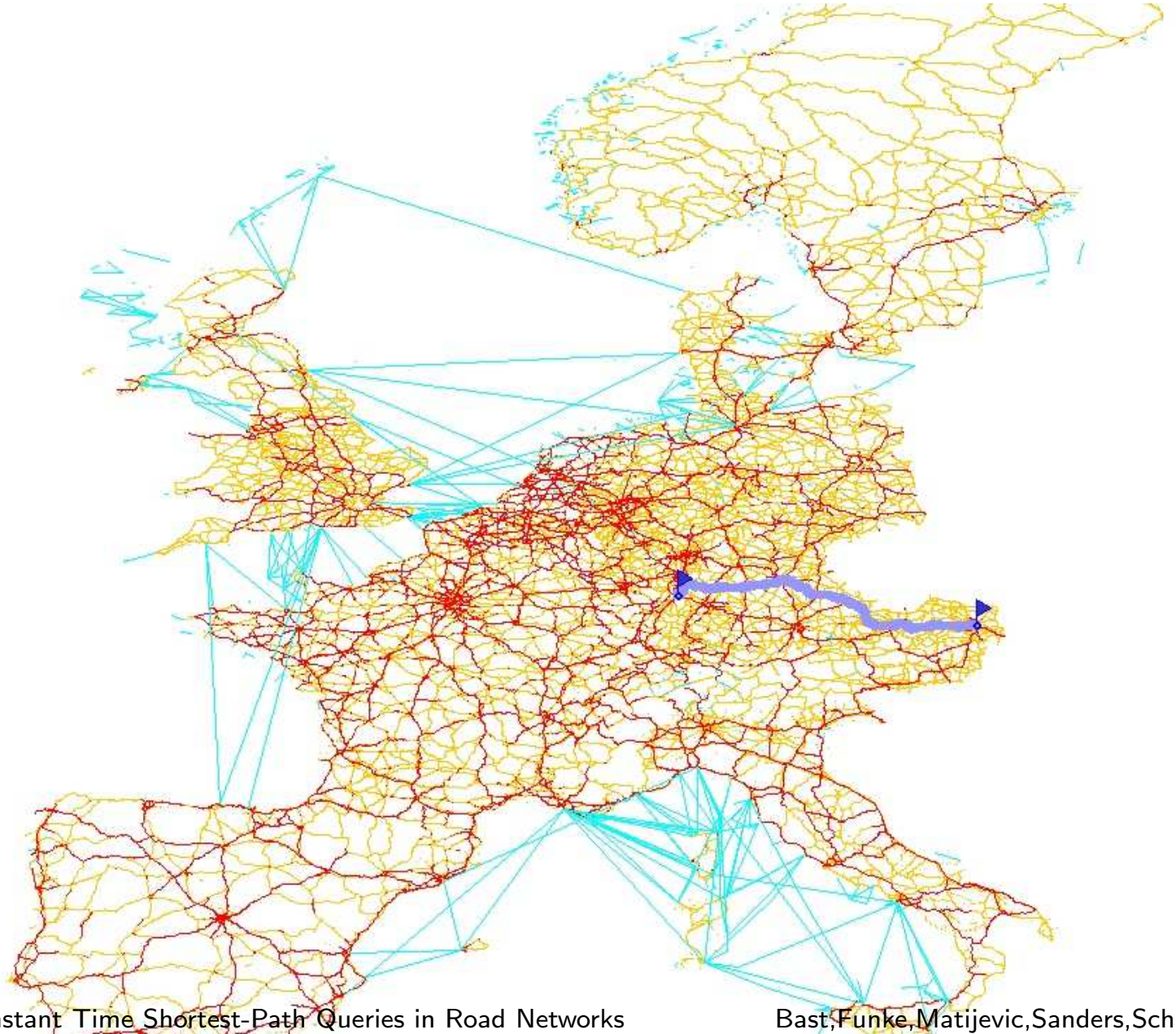

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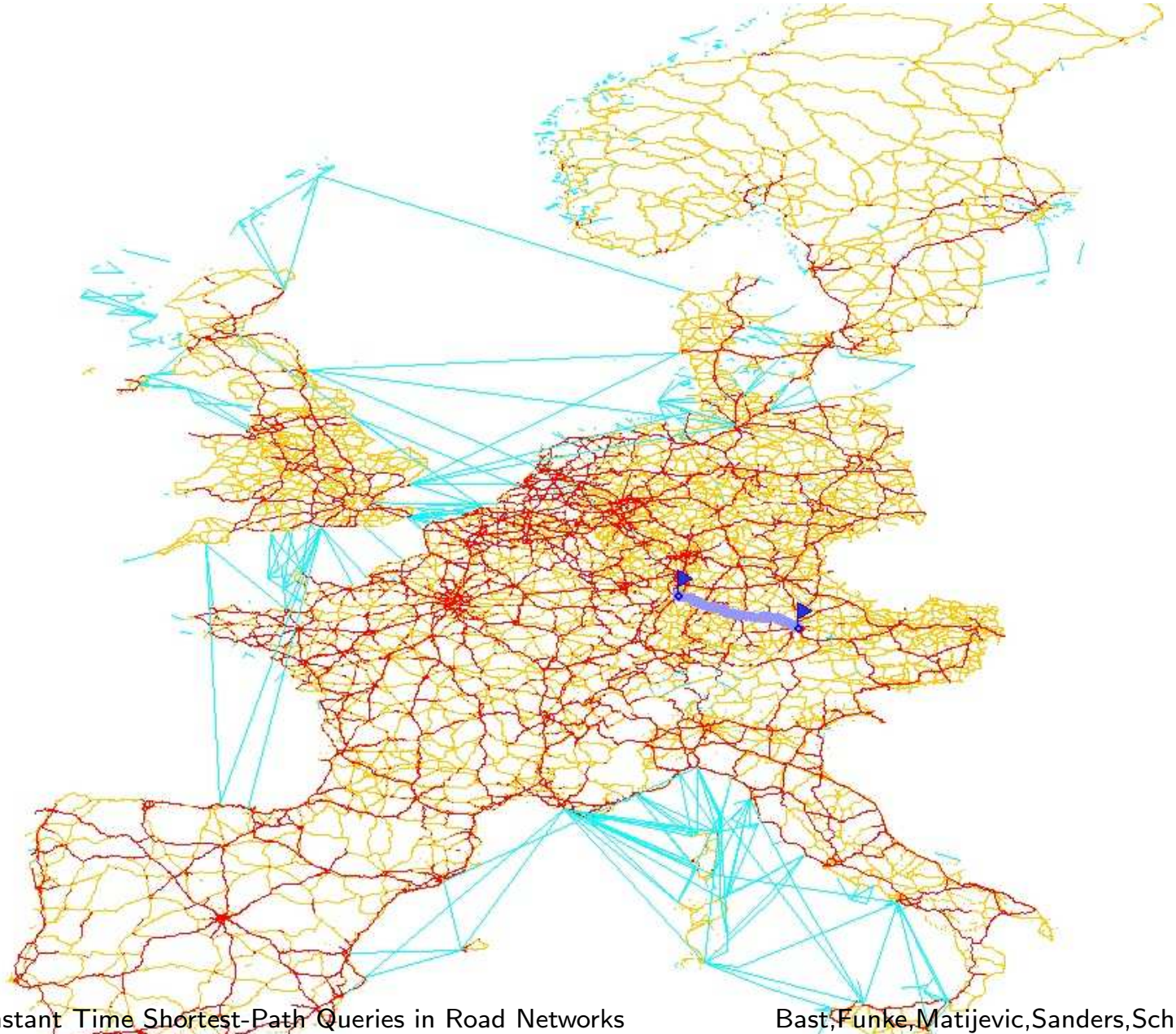

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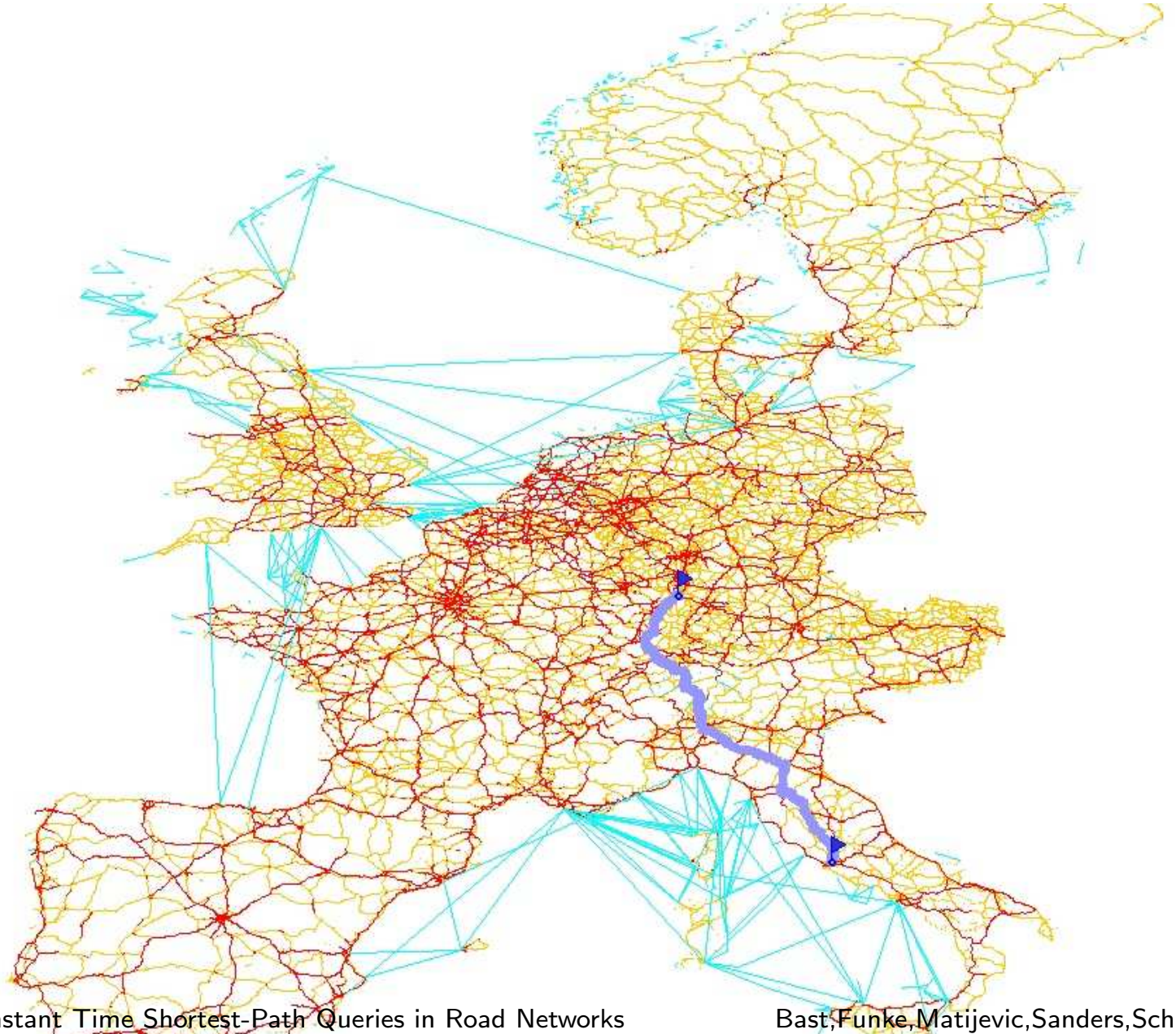

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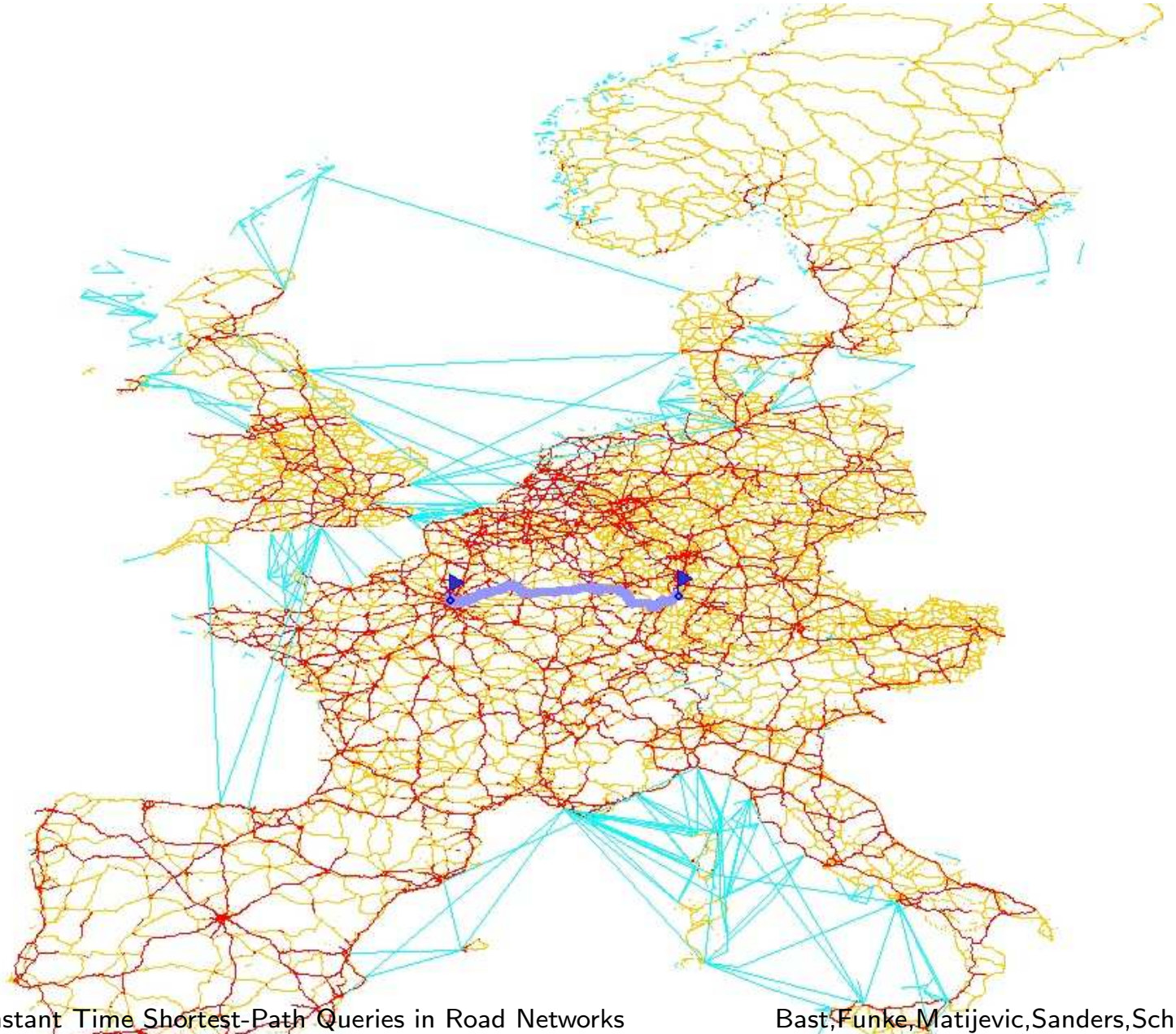

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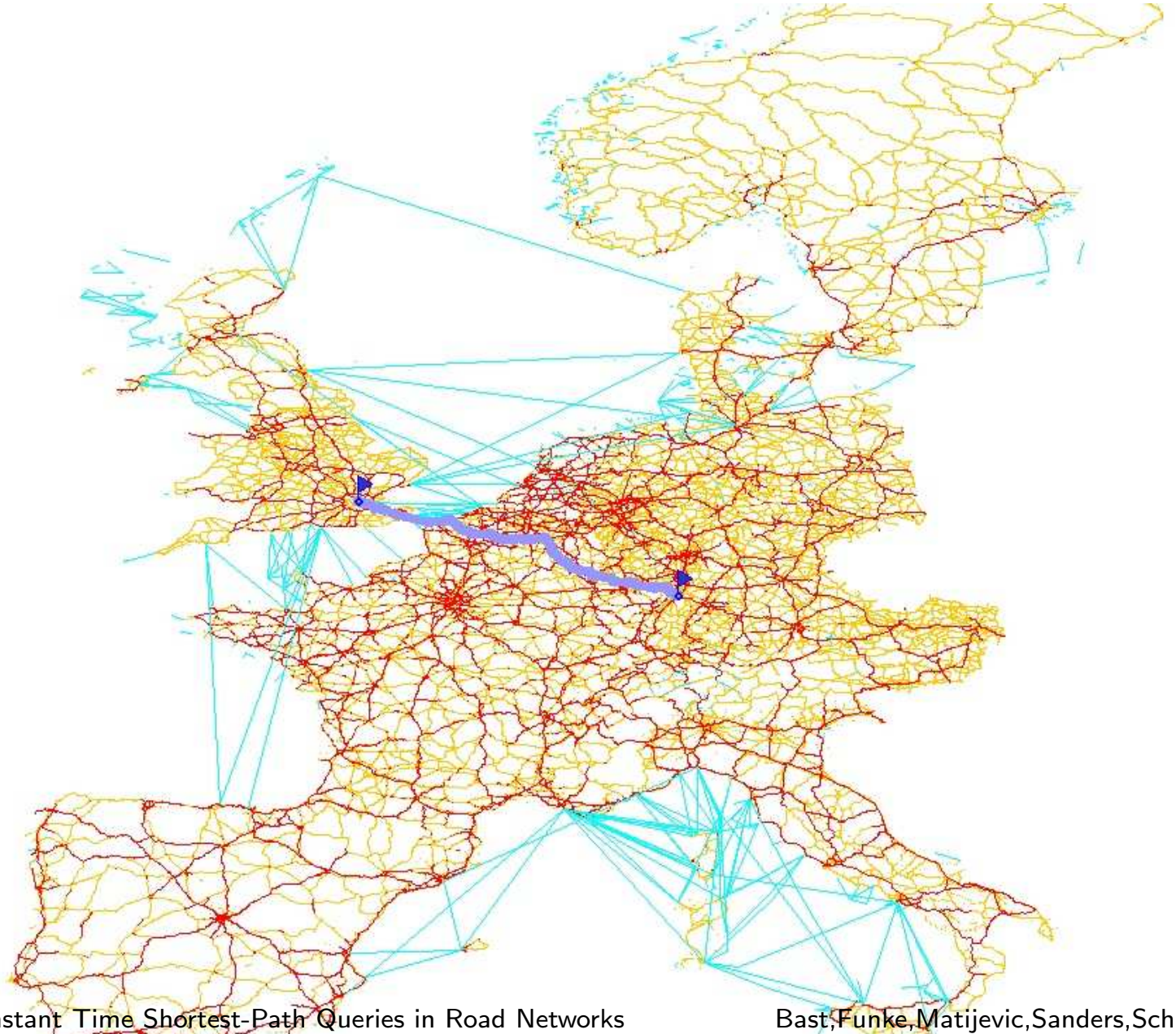

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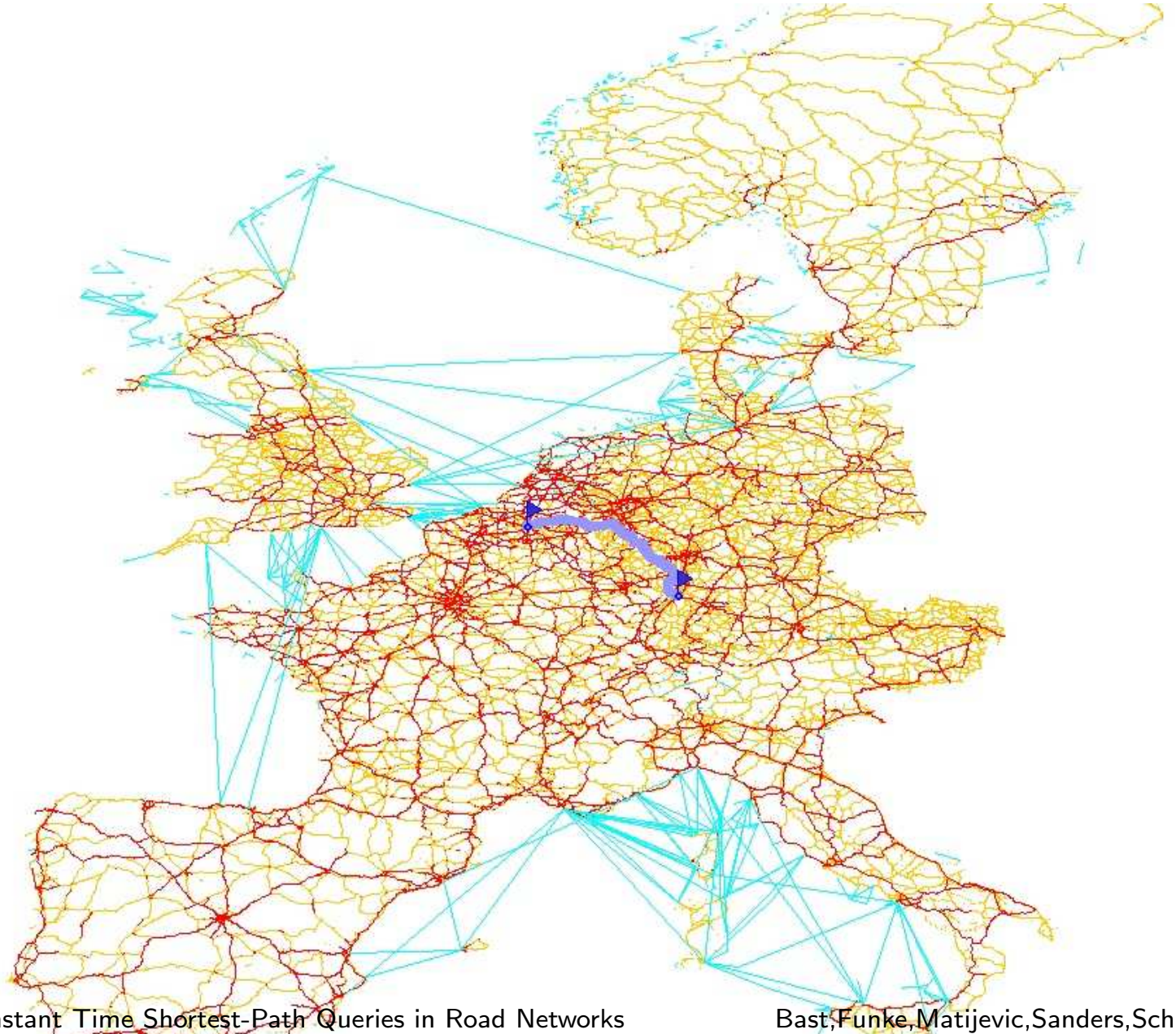

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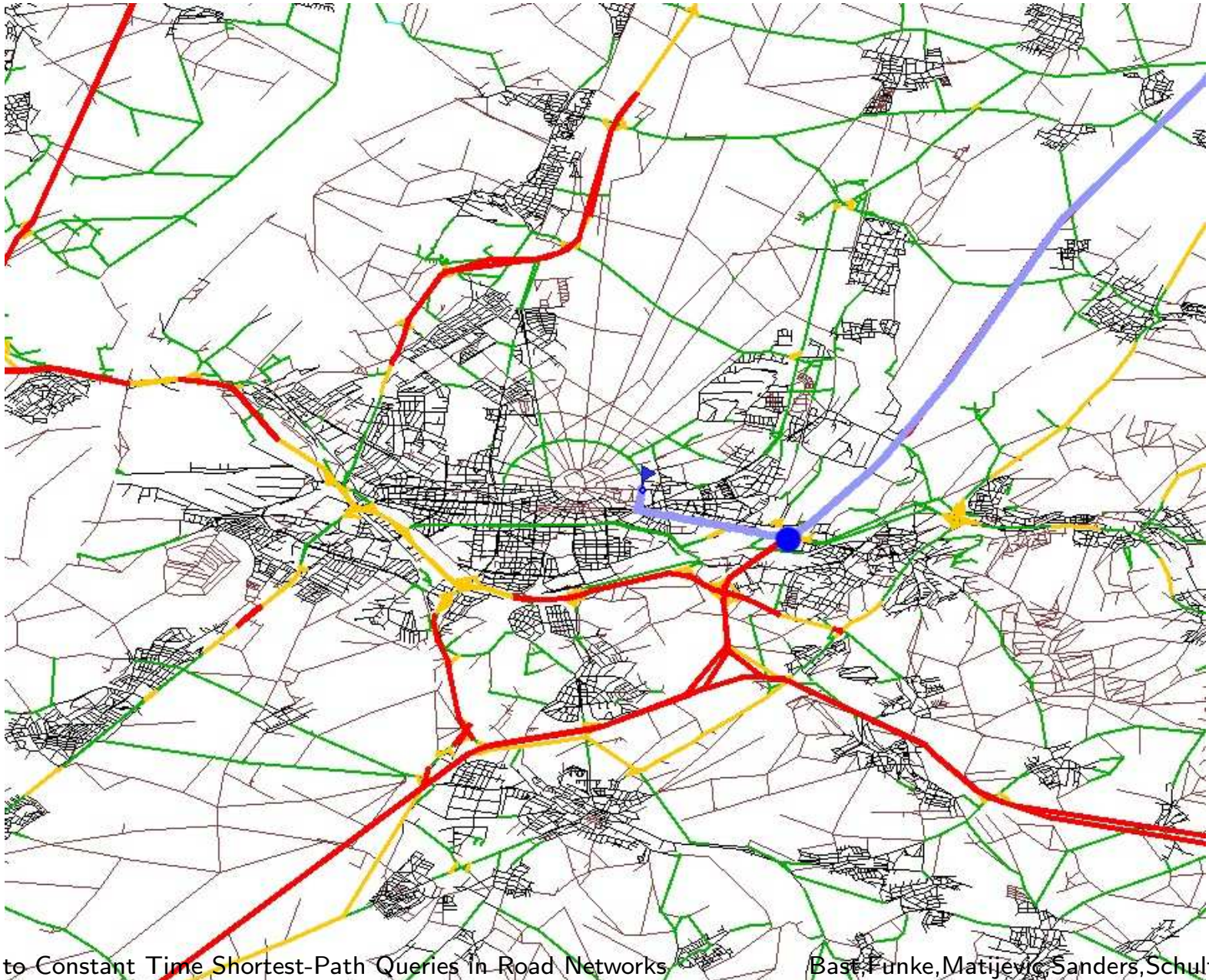

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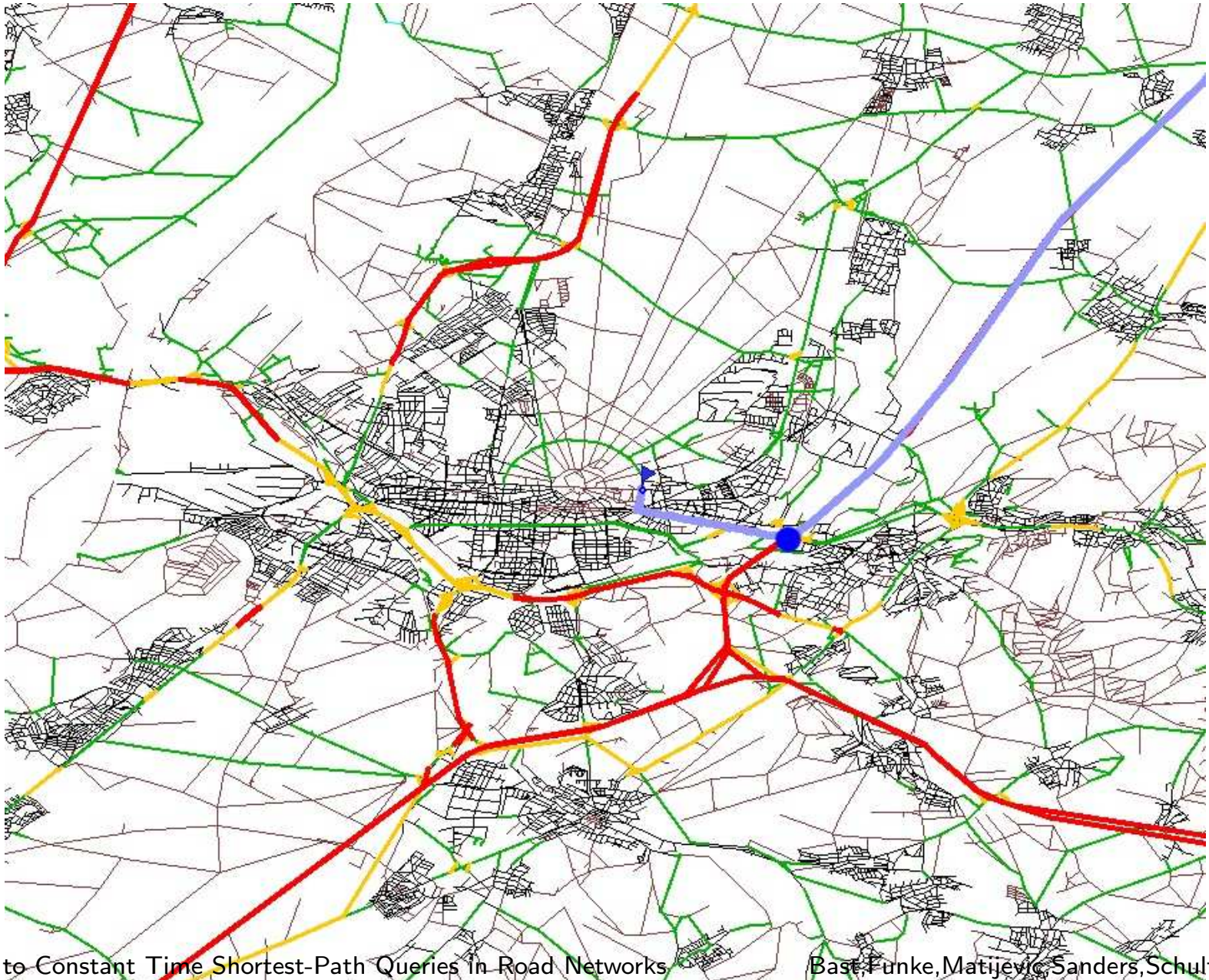

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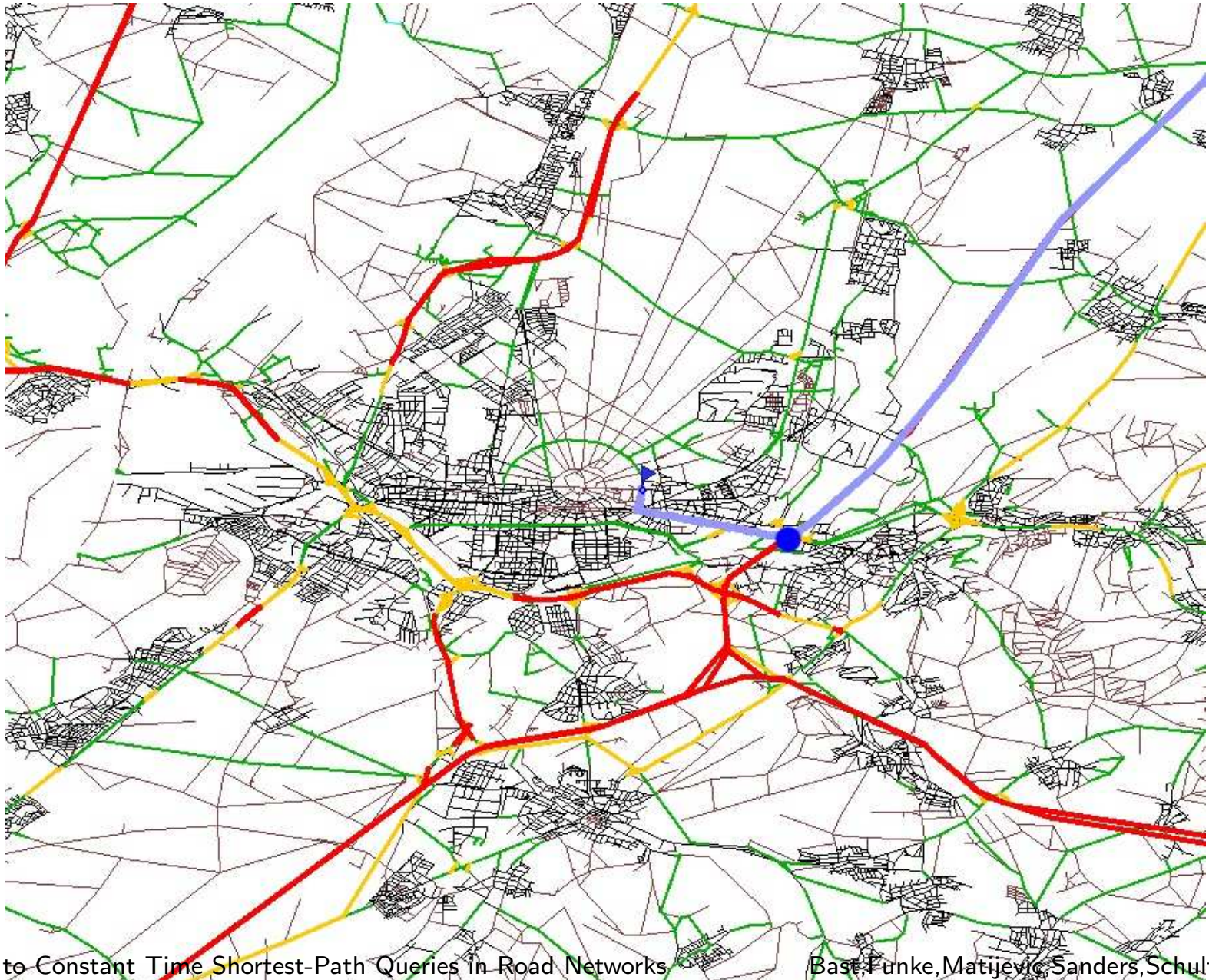

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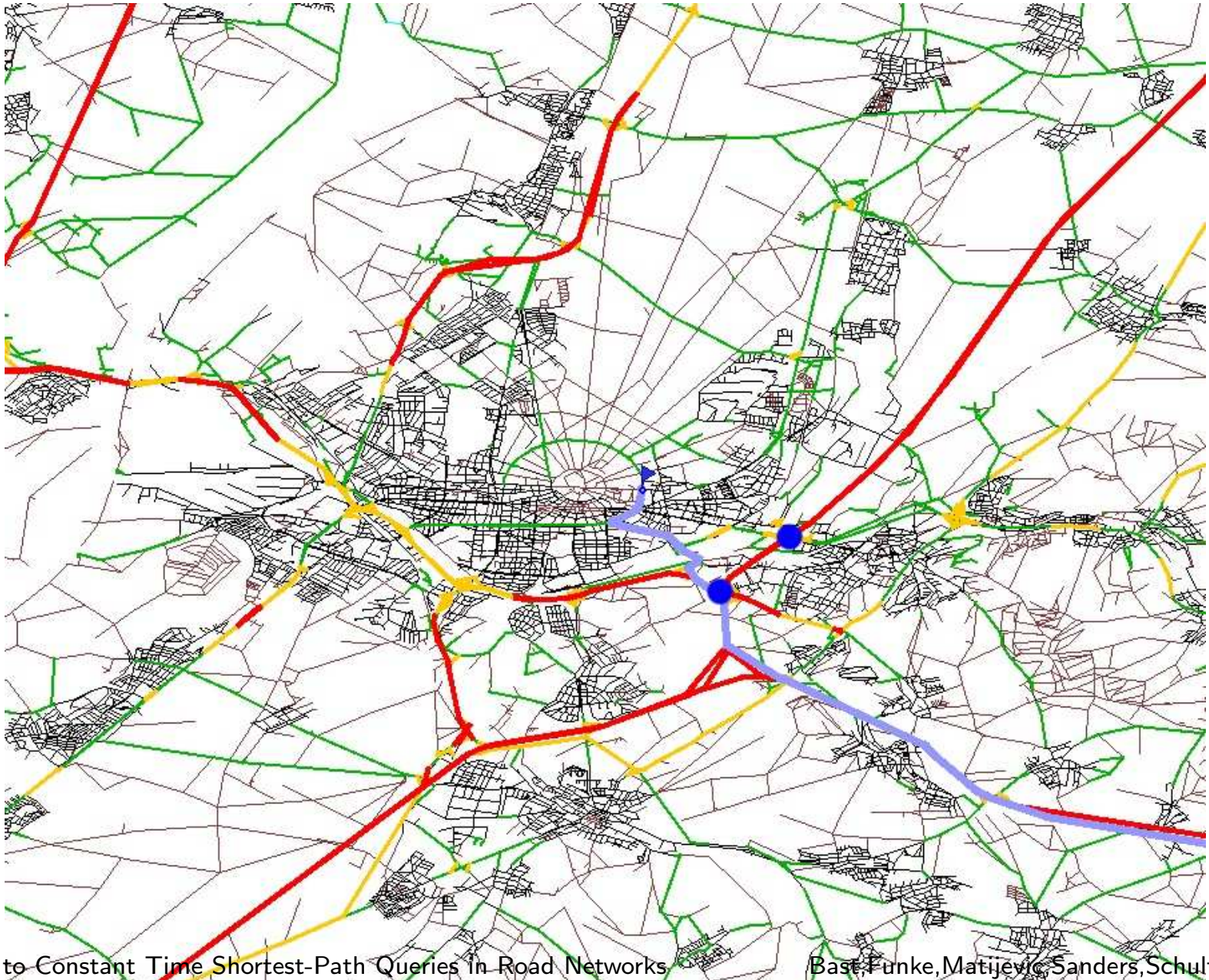

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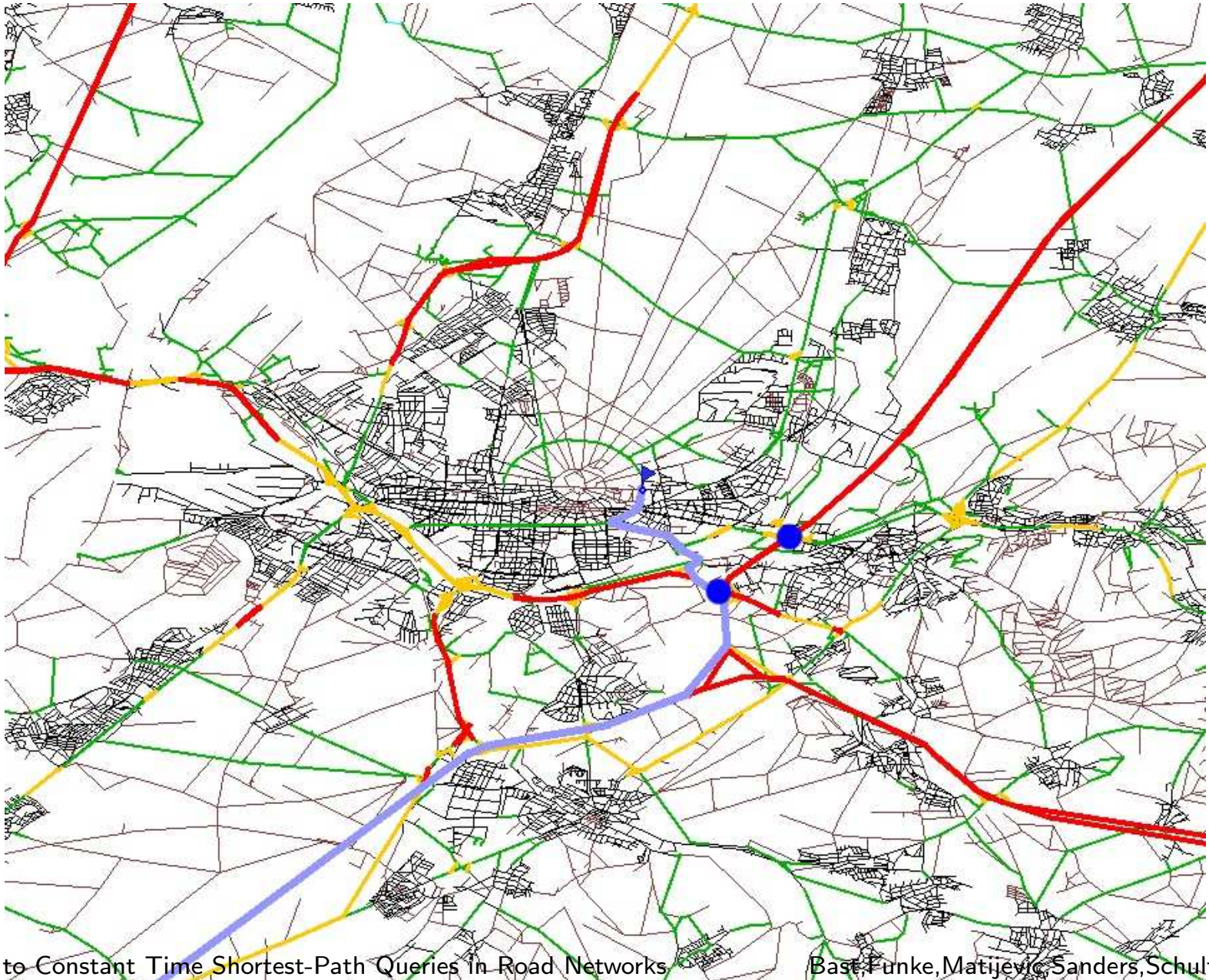

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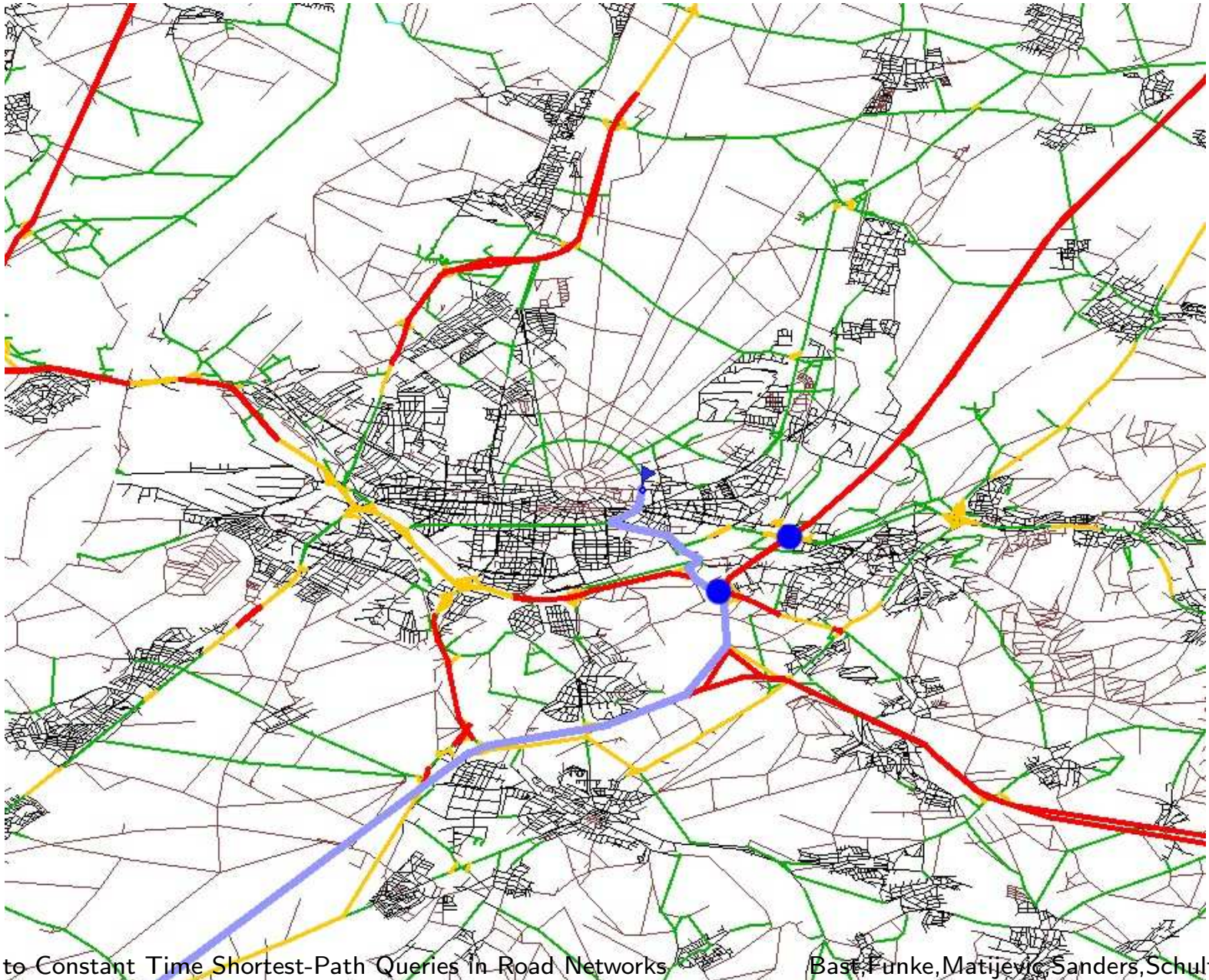

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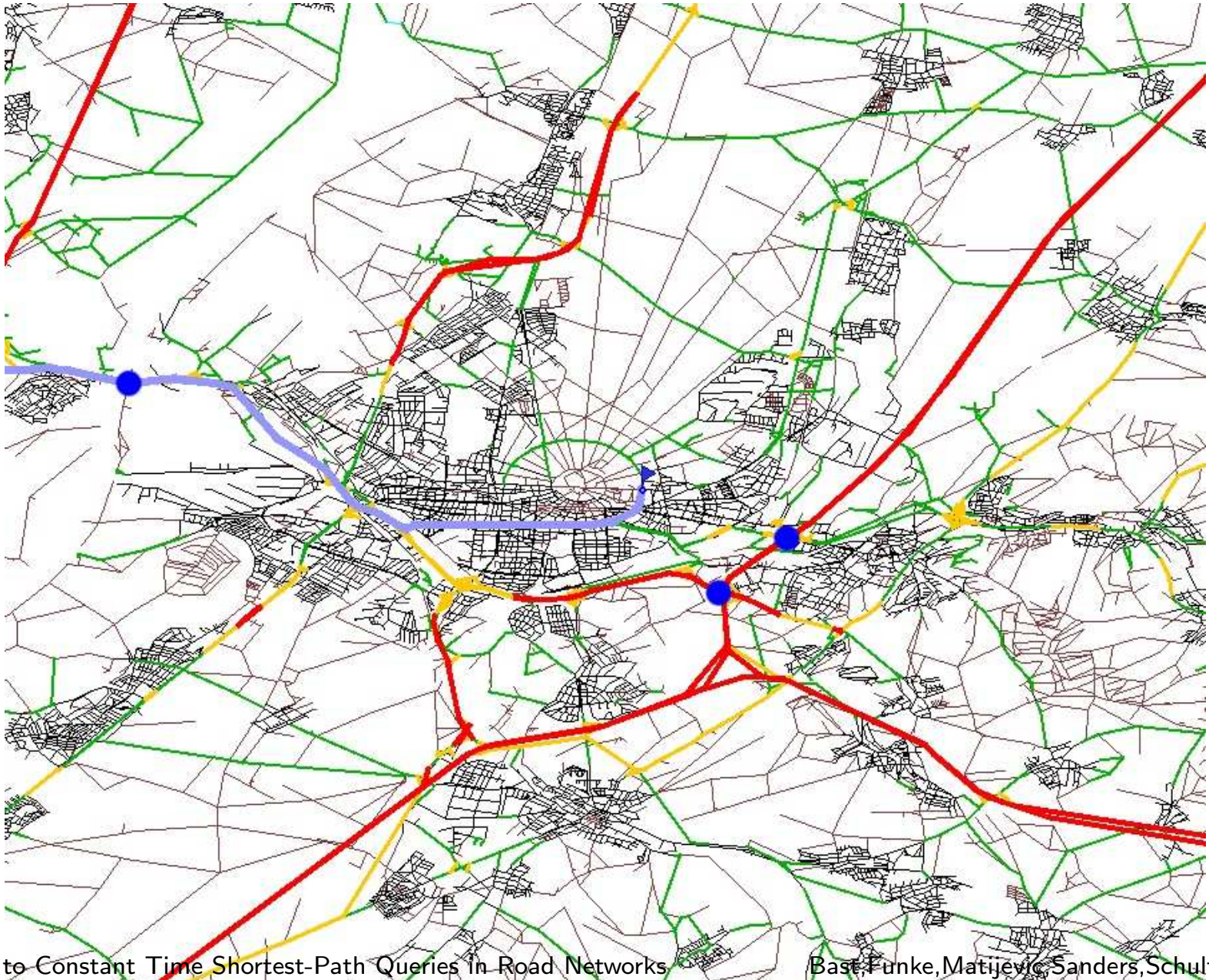

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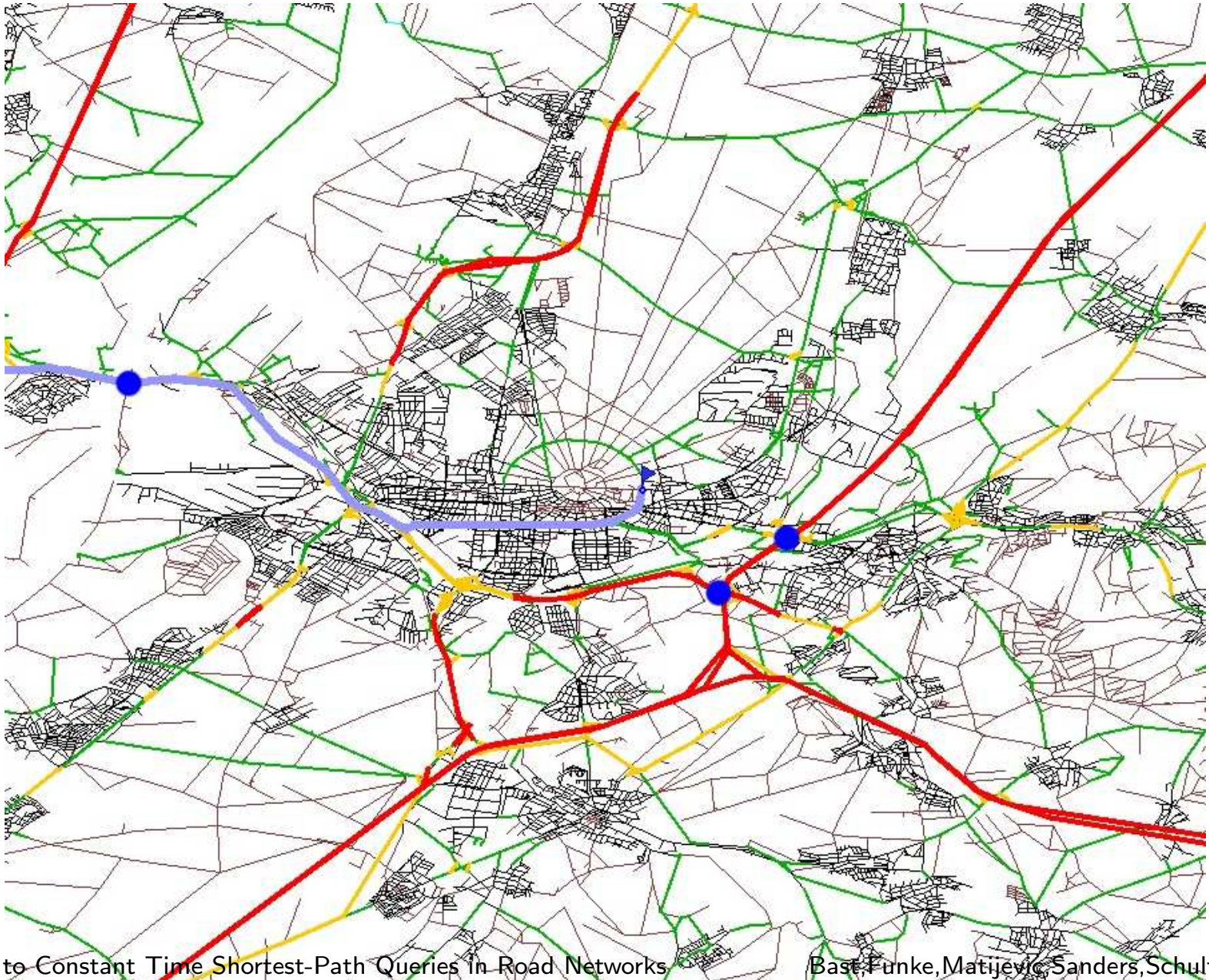

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  - a per node space overhead of 21 to 244 bytes
- $\Rightarrow$ Query times *orders of magnitudes* better than previously reported results

Milliseconds ($10^{-3}$) vs. Microseconds ($10^{-6}$)!

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What to do with the transit/access nodes ?

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- NO $\rightarrow$ use favourite SP data structure – HH, edge reach ...

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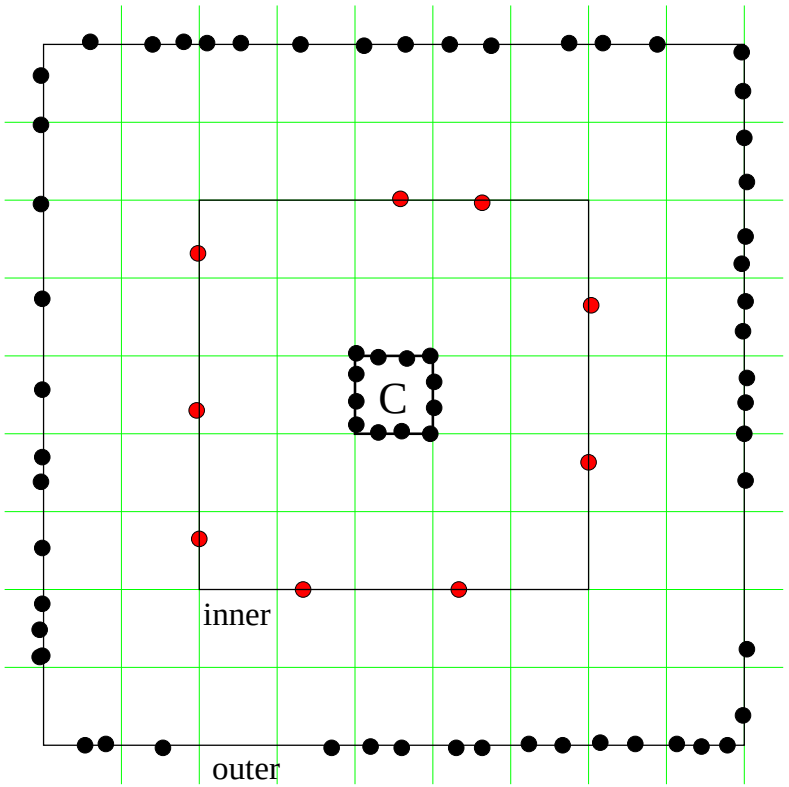
First attempt – adhoc realization of the core idea.

- impose a grid, e.g. $128 \times 128$ over the network



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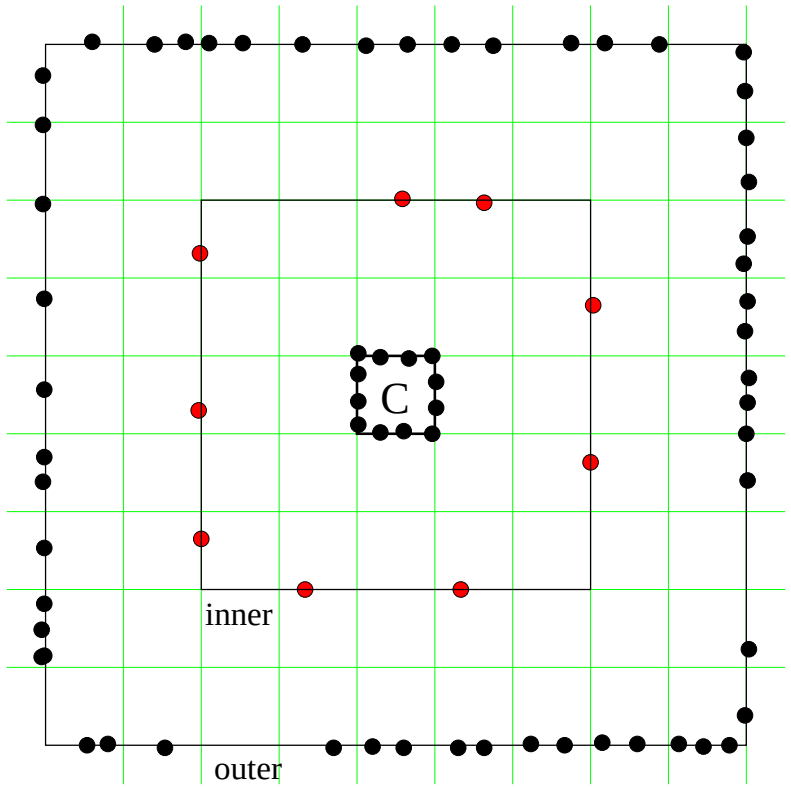

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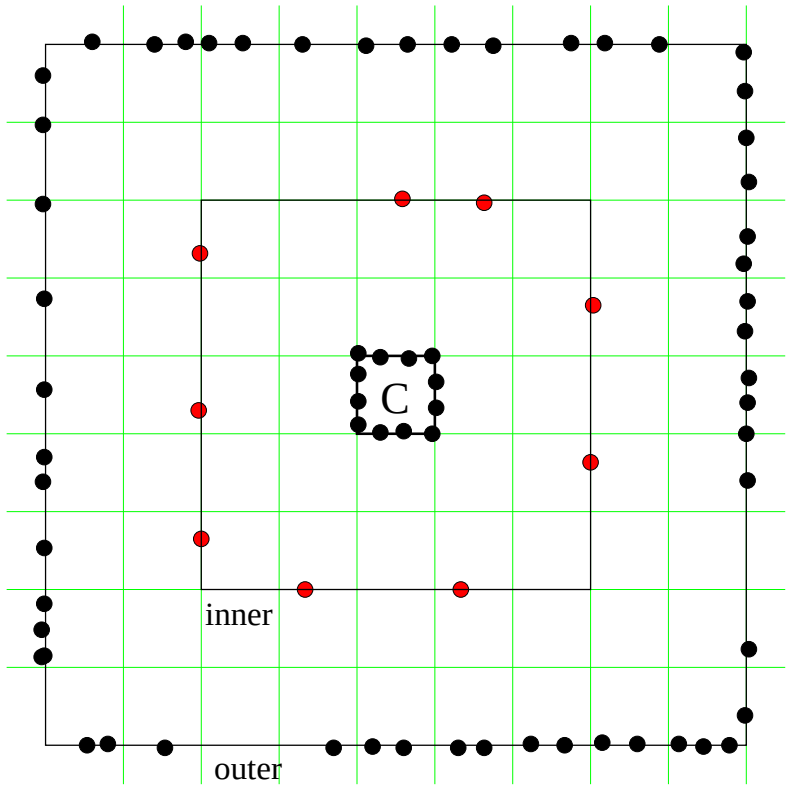

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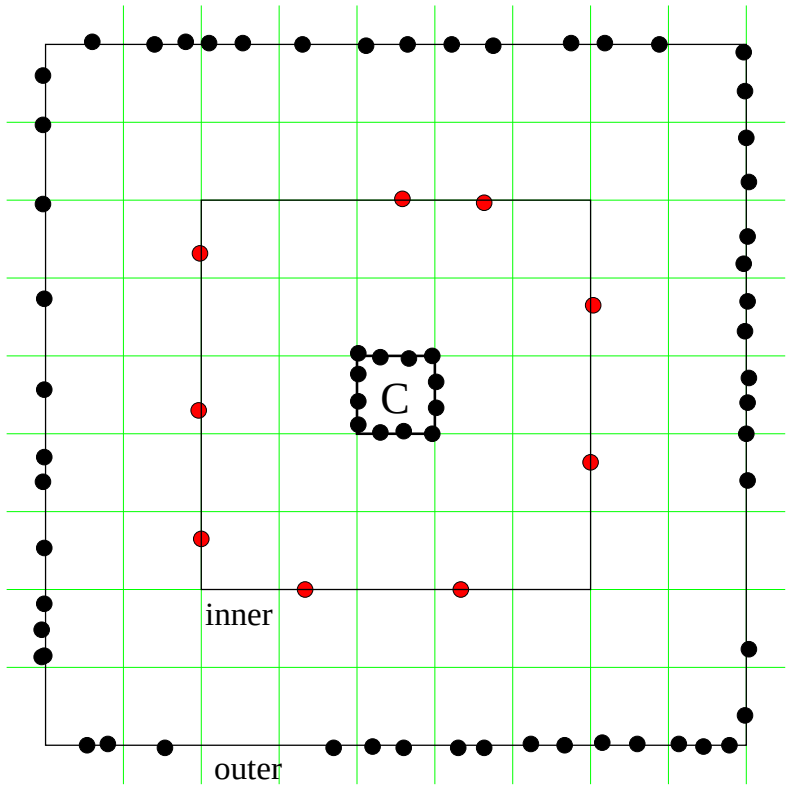

# Grid-based Implementation: What are 'long' paths?

Path/Query between source $s$ and target $t$ 'long'/non-local

$\Leftrightarrow$

$s$ and $t$ at least 4 grid cells (horiz./vert.) apart

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- multi-layered implementation also possible
- can be made very space-efficient

# Experiments: US roadmap ($n = 24$ Mio, $m = 58$ Mio)

| Grid | $\lvert\mathcal{T}\rvert$ | $\frac{\lvert\mathcal{T}\rvert \times \lvert\mathcal{T}\rvert}{\text{node}}$ | avg. $\lvert\mathcal{A}\rvert$ | % 'long' queries | construction of transit nodes |
|---|---|---|---|---|---|
| $64 \times 64$ | 2 042 | 0.1 | 11.4 | 91.7% | 498 min |
| **$128 \times 128$** | **7 426** | **1.1** | **11.4** | **97.4%** | **525 min** |
| **$256 \times 256$** | **24 899** | **12.8** | **10.6** | **99.2%** | **638 min** |
| $512 \times 512$ | 89 382 | 164.6 | 9.7 | 99.8% | 859 min |
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Statistics for several grid sizes.

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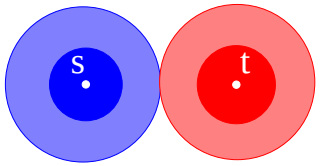
Statistics for several grid sizes.

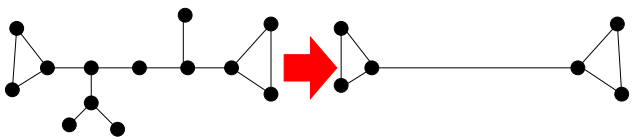
| non-local (99%) | local (1%) | all queries | preproc. | space/node |
|---|---|---|---|---|
| $12\mu s$ | $5112\mu s$ | $63\mu s$ | 20h | **21 bytes** |

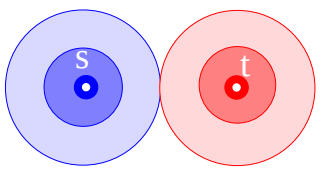
Results for 2-layer data structure.

# Highway Hierarchies [Sanders/Schultes ESA'05/'06]




- Gutman's insight with 2nd metric = Dijkstra rank
- complete search within a local area
- identify *highway network* = minimal subgraph that preserves all 'long' shortest paths
- contract network
- iterate $\Rightarrow$ *highway hierarchy*





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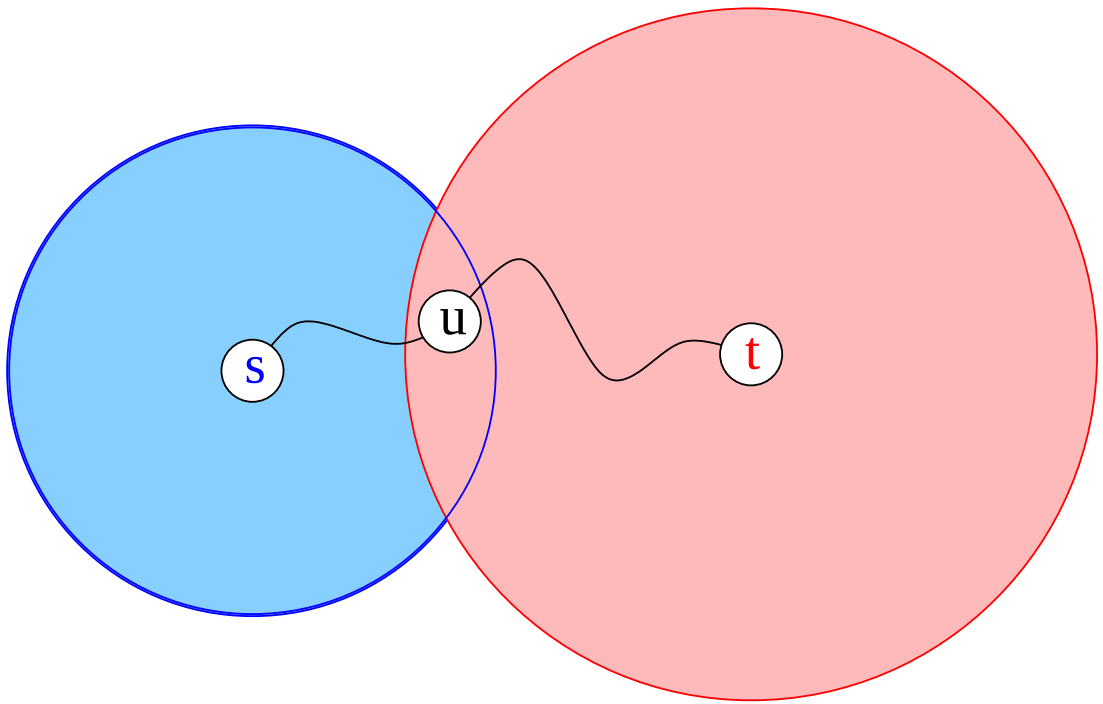
  $\Rightarrow$ high level in HH canonical choice for transit nodes $\mathcal{T}$
- Using several levels from HH induce multi-layer solution in a very natural way
- Local queries can be handled very efficiently by HH ($\Leftrightarrow$ grid-based approach)

# HH-based Transit Node Routing: What are 'long' paths?

Compute for each node $v$ a radius $r(v)$ such that a query $(s, t)$ is non-local/the path is considered long, if respective disks with radii $r(s)$ and $r(t)$ do **not** overlap.



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- A more economical version in terms of storage space and preprocessing times uses levels 5 and 3 in a 2-layer structure
- Implemented and benchmarked also for directed graphs, e.g. roadmap of Europe, and *distances* instead of travel times

# Experiments: US roadmap ($n = 24$ Mio, $m = 58$ Mio)

Preprocessing:

| | variant | layer 1 $\lvert\mathcal{T}\rvert$ | $\lvert\mathcal{A}\rvert$ | layer 2 $\lvert\mathcal{T}_2\rvert$ | $\lvert\mathcal{A}_2\rvert$ | space [B/node] | time [h] |
|---|---|---|---|---|---|---|---|
| USA | eco | 12 111 | 6.1 | 184 379 | 4.9 | 111 | *0:59* |
| | gen | 10 674 | 5.7 | 485 410 | 4.2 | 244 | 3:25 |

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Query:

| | variant | layer 1 [%] wrong | layer 1 [%] cont'd | layer 2 [%] wrong | layer 2 [%] cont'd | time |
|---|---|---|---|---|---|---|
| | eco | 0.14 | 1.13 | 0.0064 | 0.2780 | 11.5 $\mu$s |
| USA | gen | 0.11 | 0.80 | 0.0014 | 0.0138 | *4.9 $\mu$s* |

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  - simple, grid-based implementation tuned for space efficiency ($63\mu s$ avg.query; 20h preprocessing; 21 bytes/node)
  - sophisticated, extremely fast implementation based on HH ($5\mu s$; $< 4$h; 244 bytes/node); fast for *all* types of queries

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- not mentioned: efficient ways to output actual paths
- room for improvement in both realizations: grid-based in terms of query/preprocessing, HH-based in terms of space consumption
- One of the main challenges: deal with dynamics of real-world networks

# Thank you for your attention!