Engineering Route Planning Algorithms

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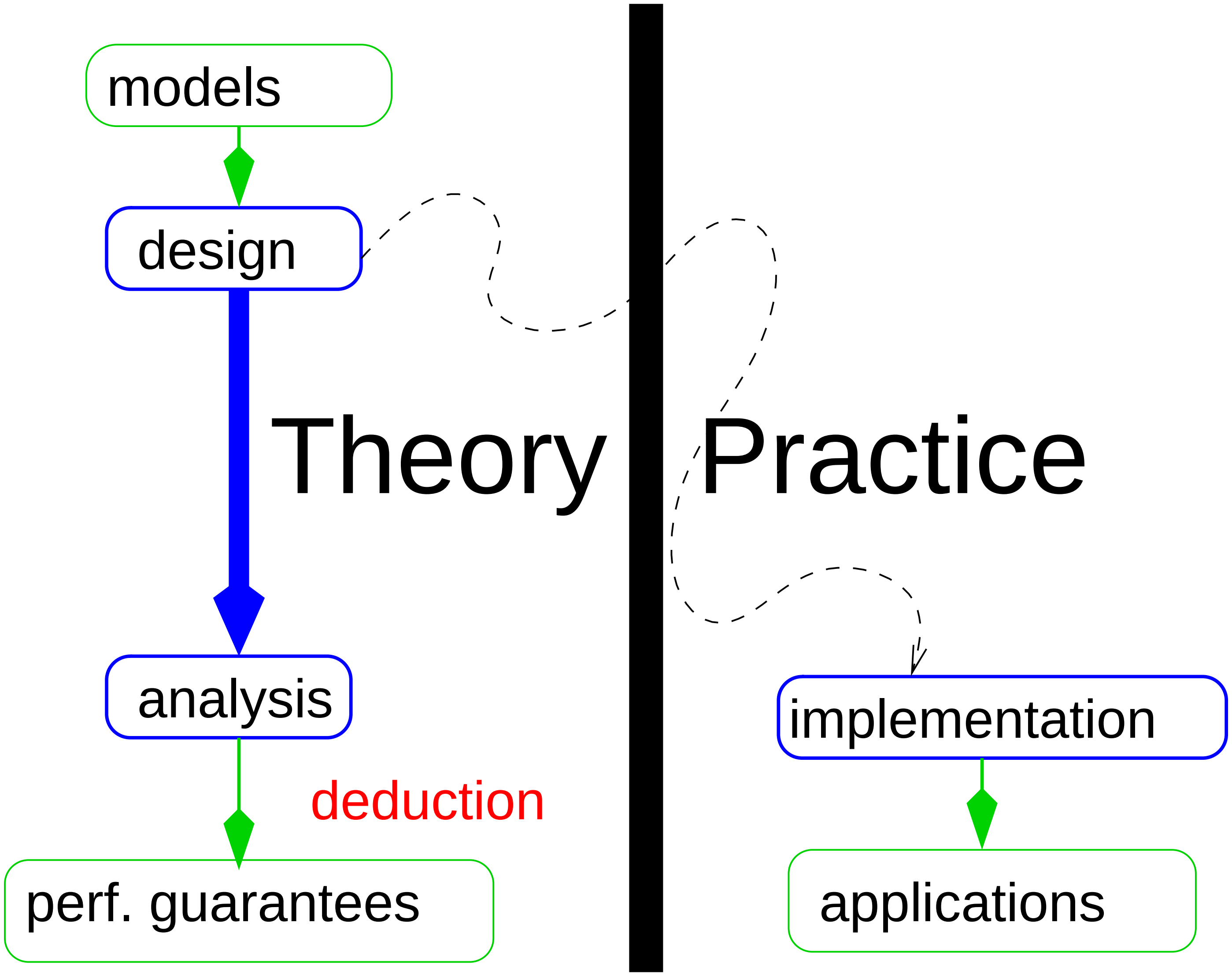
`http://algo2.iti.uka.de/schultes/hwy/`

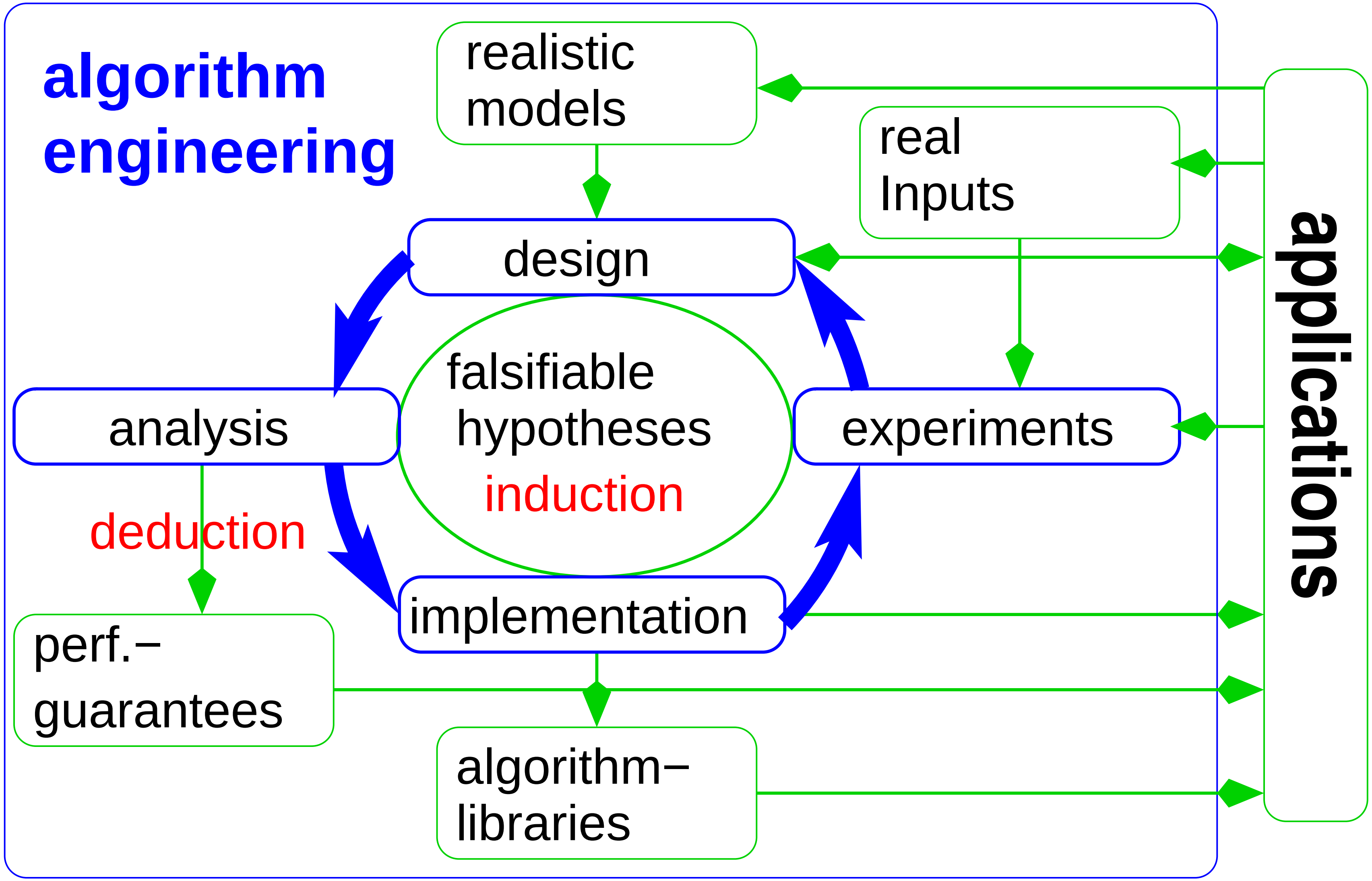
Overview

- Algorithm Engineering
- Route Planning
- Related Work — fast
- Highway Hierarchies — faster
- Many-to-Many Routing
- Transit-Node Routing — fastest
- Summary
- Future Work

(Caricatured) Traditional View: Algorithm Theory



Algorithmics as Algorithm Engineering



Route Planning: How do I get there from here ?

Applications

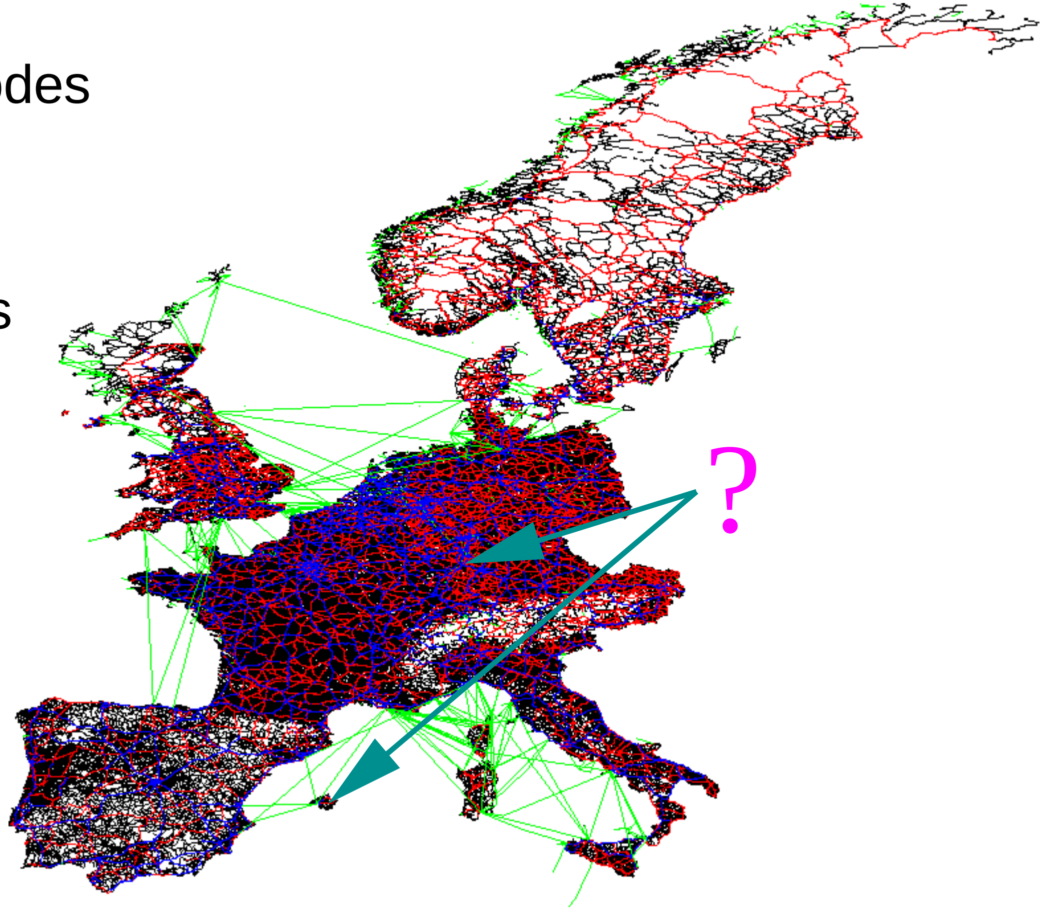
- route planning systems in the internet (e.g. `www.map24.de`)

- car navigation systems

- logistics planning
- traffic simulation

Road Networks

- Large, e.g. $n =$18 000 000 nodes for Western Europe

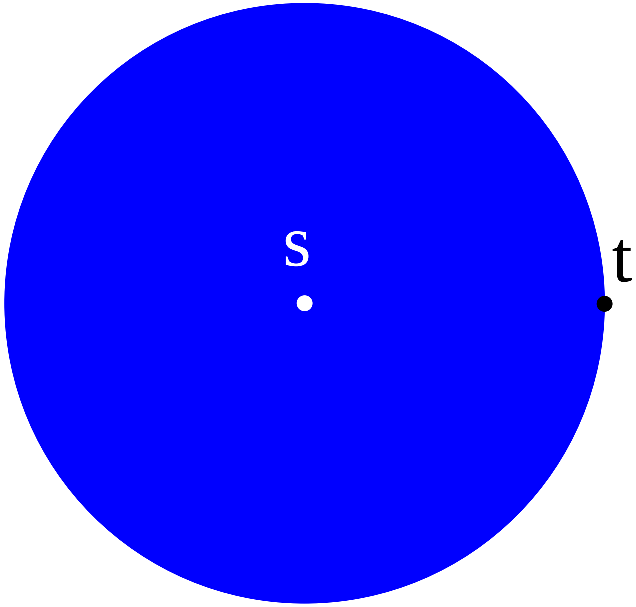


- Sparse, i.e., $m = \Theta(n)$ edges
- Almost planar, i.e., few edges cross
- Quickest paths use important streets
- Changes are slow/few, i.e., Fast, near linear space preprocessing OK

We want fast, exact, point-to-point queries

DIJKSTRA's Algorithm

Dijkstra



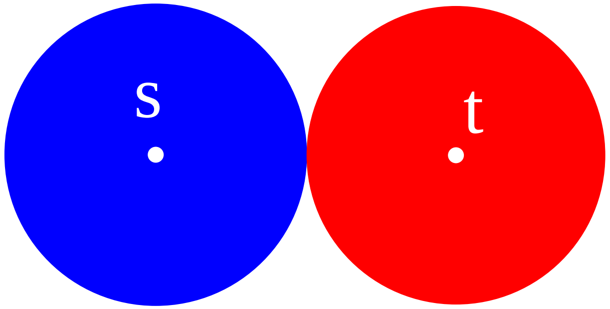
not practicable

for large road networks

(e.g. Western Europe:

≈ 18 000 000 nodes)

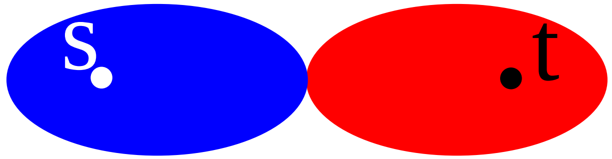
bidirectional Dijkstra



improves the running time,

but still too slow

Goal-Directed Search



A^* [Hart, Nilsson, Raphael 68]: not effective for travel time

Geometric Containers [Wagner et al. 99–05]:

high speedup but quadratic preprocessing time

Landmark A^* [Goldberg et al. 05–]: precompute distances to $\approx$ 20

landmarks $\leadsto$ moderate speedups, preprocessing time, space

Precomputed Cluster Distances [S, Maue 06]:

more space-efficient alternative to landmarks

Hierarchical Methods

Planar graph (theory) [Fakcharoenphol, Rao, Klein 01–06]: $O(n \log^2 n)$
space and preprocessing time; $O(\sqrt{n} \log n)$ query time

Planar approximate (theory) [Thorup 01]: $O((n \log n)/\varepsilon)$ space and
preprocessing time; almost constant query time

Separator-based multilevel [Wagner et al. 99–]:
works, but does not capitalize on importance induced hierarchy

Reach based routing [Gutman 04]:
elegant, but initially not so successful

Highway hierarchies [SS 05–]: stay tuned

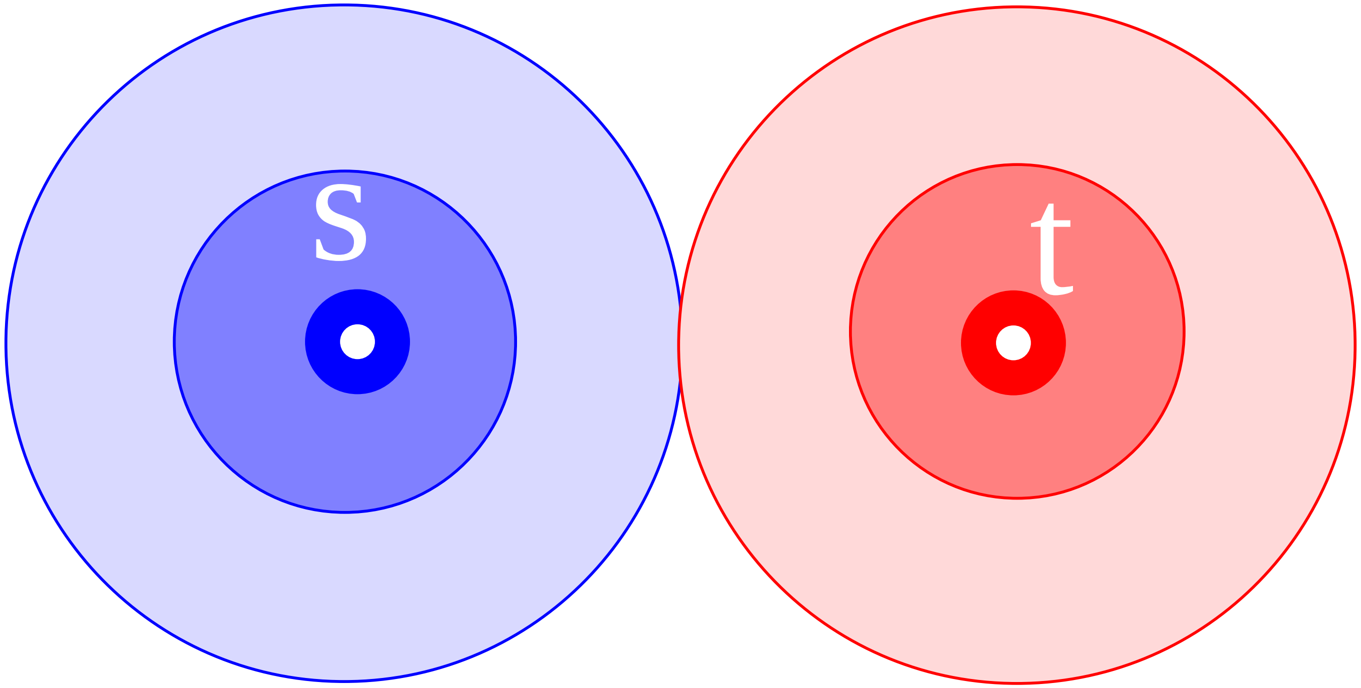
Advanced reach [Goldberg et al. 06–]: combinable with landmark A^*

Transit-node routing [Bast, Funke, Matijevic, S, S 07–]: stay tuned

Highway-node routing [SS 07–]: stay tuned

Highway Hierarchies

[SS 05–]



Naive Route Planning

1. Look for the next reasonable motorway

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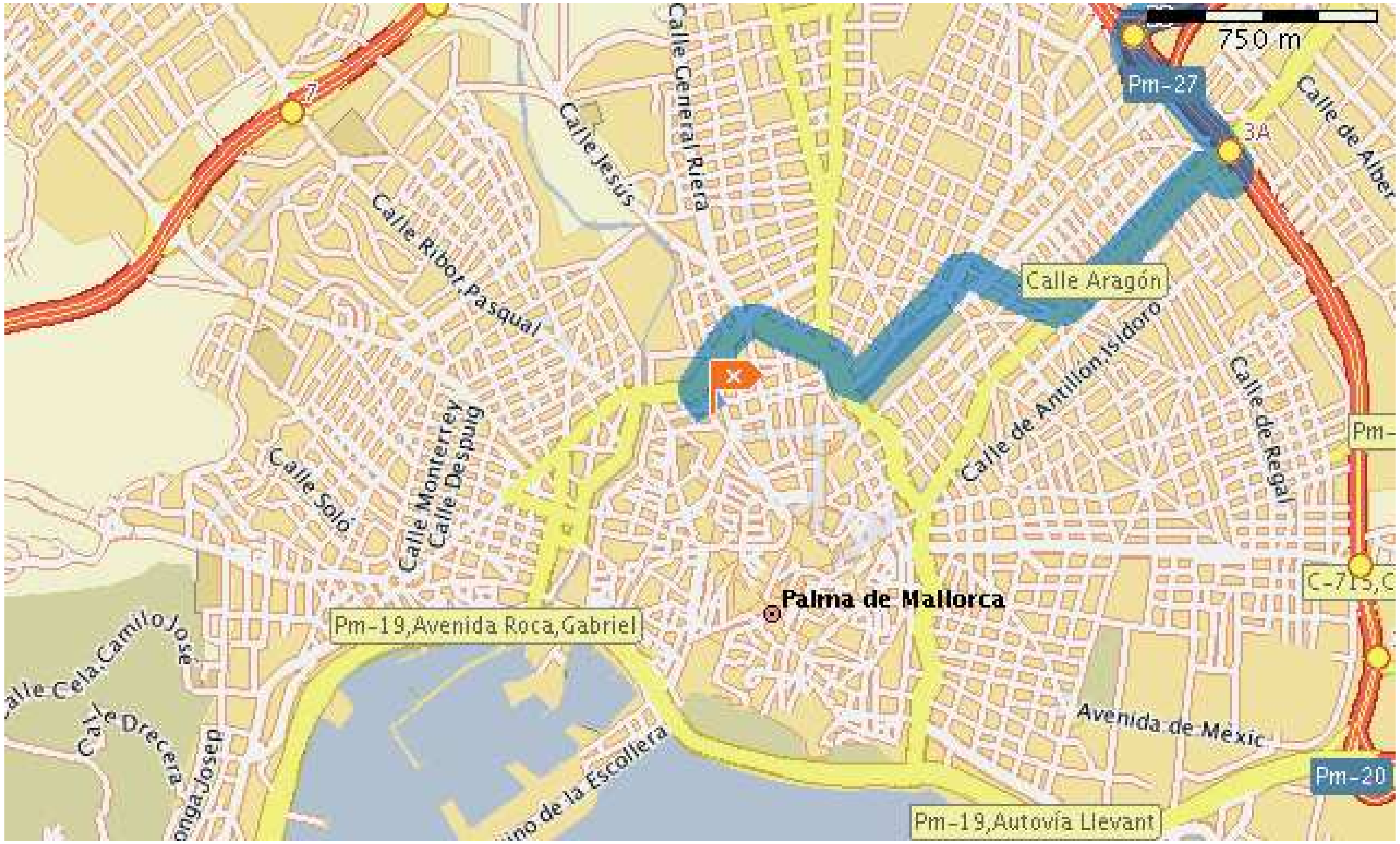
2. Drive on motorways to a location close to the target

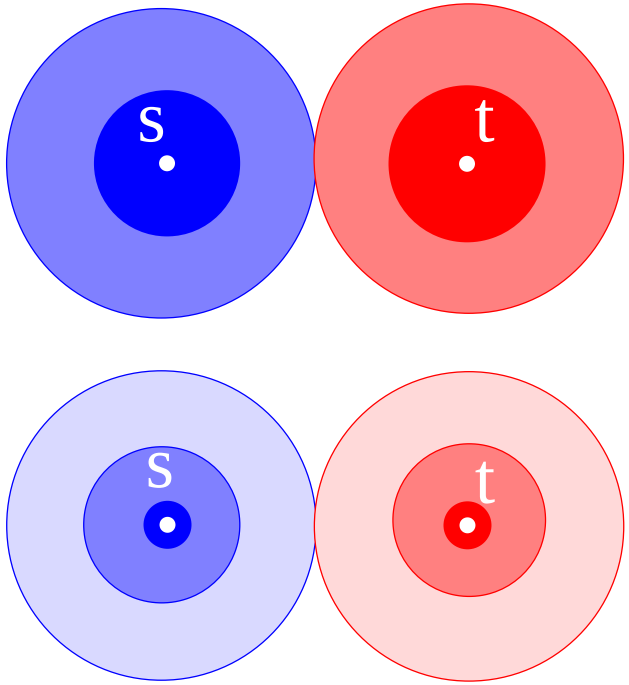
Naive Route Planning

1. Look for the next reasonable motorway
2. Drive on motorways to a location close to the target
3. Search the target starting from the motorway exit



Commercial Approach

Heuristic Highway Hierarchy



- complete search in local area
- search in (sparser) highway network
- iterate ⇝ highway hierarchy

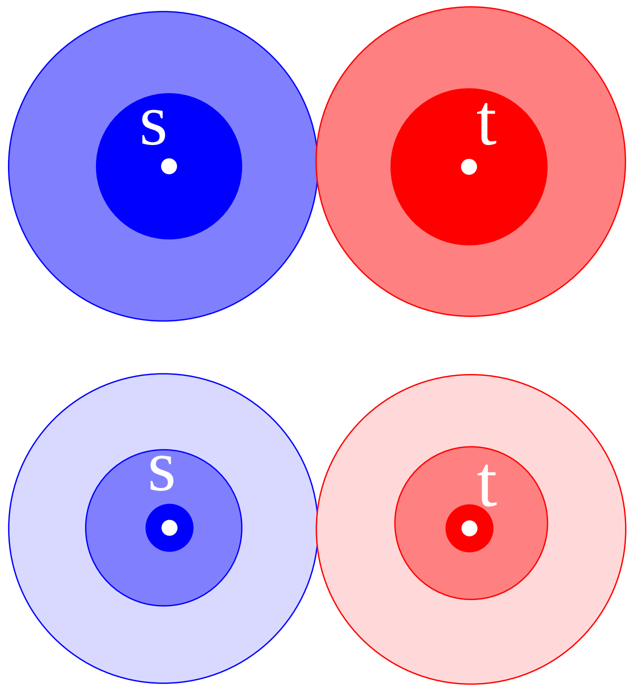
Defining the highway network:

use road category (highway, federal highway, motorway,. . .)

+ manual rectifications

- delicate compromise
- **speed ⇔ accuracy**

Our Approach



Exact Highway Hierarchy

- complete search in local area
- search in (sparser) highway network
- iterate ⇝ highway hierarchy

<u>Defining the highway network:</u>

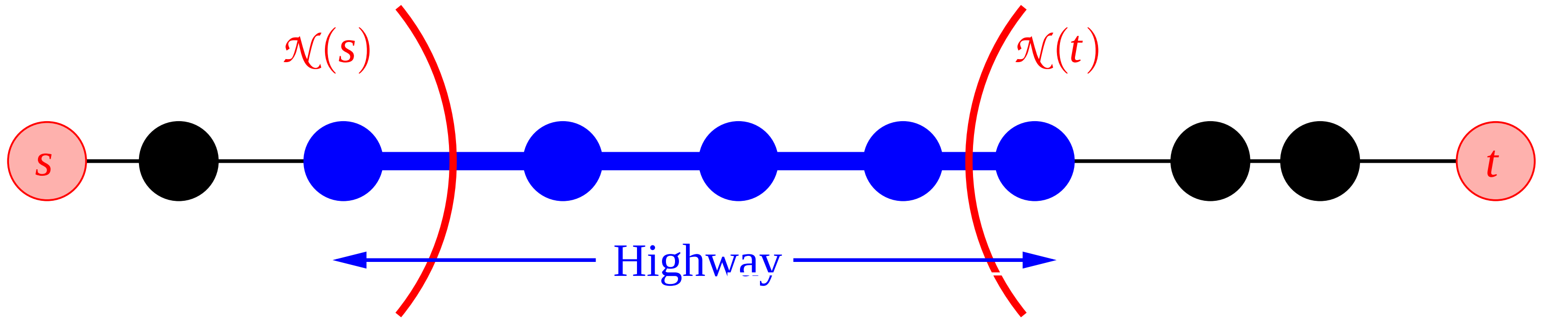
minimal network that preserves all shortest paths

- fully automatic (just fix neighborhood size)
- uncompromisingly **fast**

Constructing Exact Highway Hierarchies

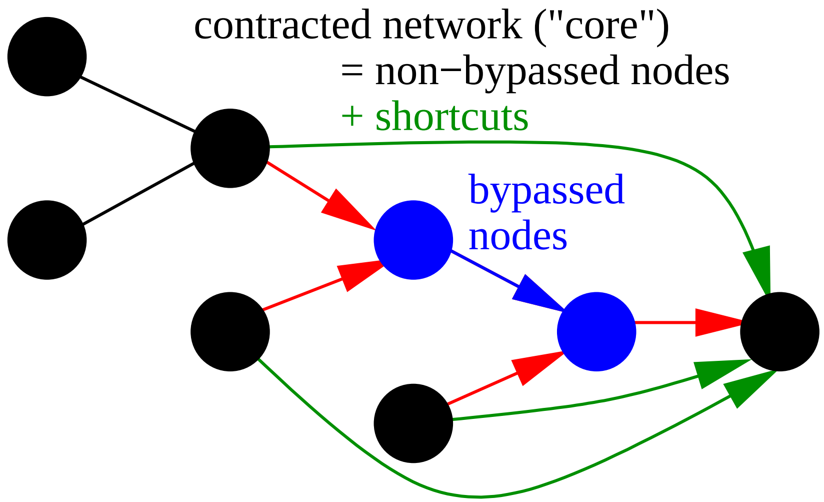
Alternate between two phases:

Edge reduction to highway edges needed outside local searches.

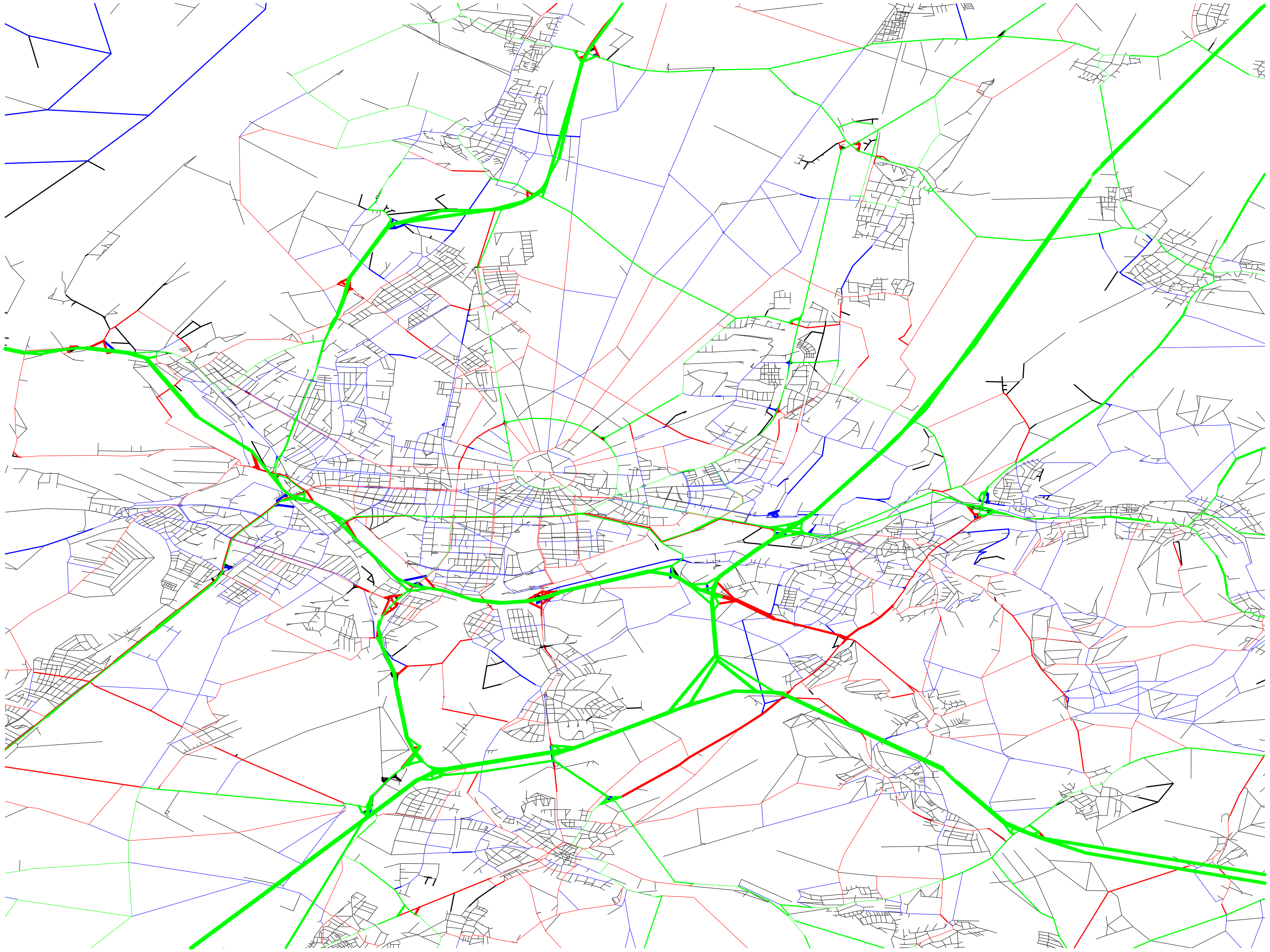


Node reduction.

Remove low degree nodes



Example: Karlsruhe



Query

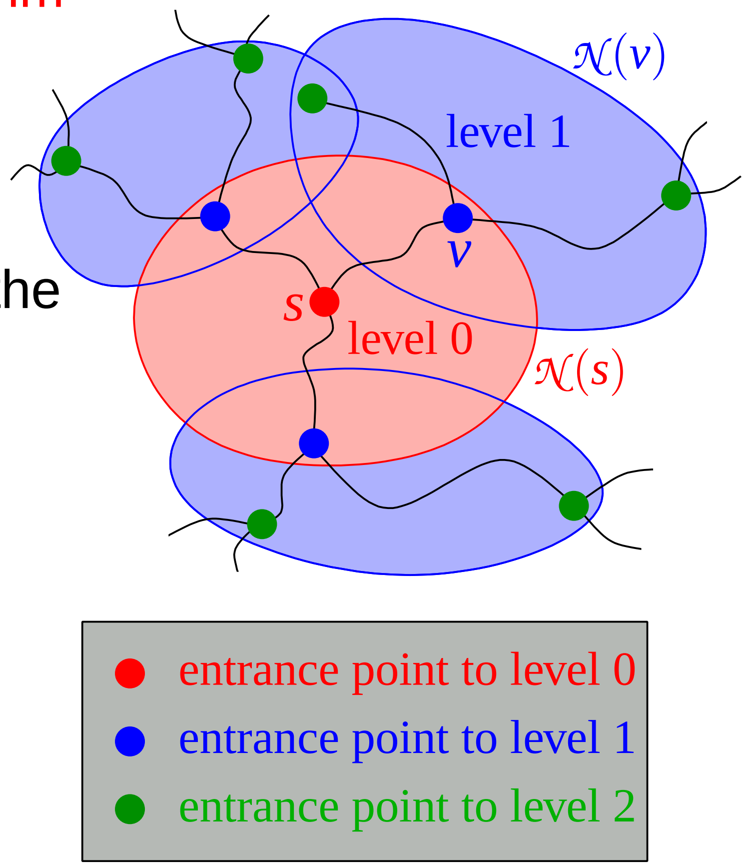
Bidirectional version of Dijkstra's Algorithm

Restrictions:

- Do not leave the neighbourhood of the entrance point to the current level.

  Instead: switch to the next level.

- Do not enter a component of bypassed nodes.



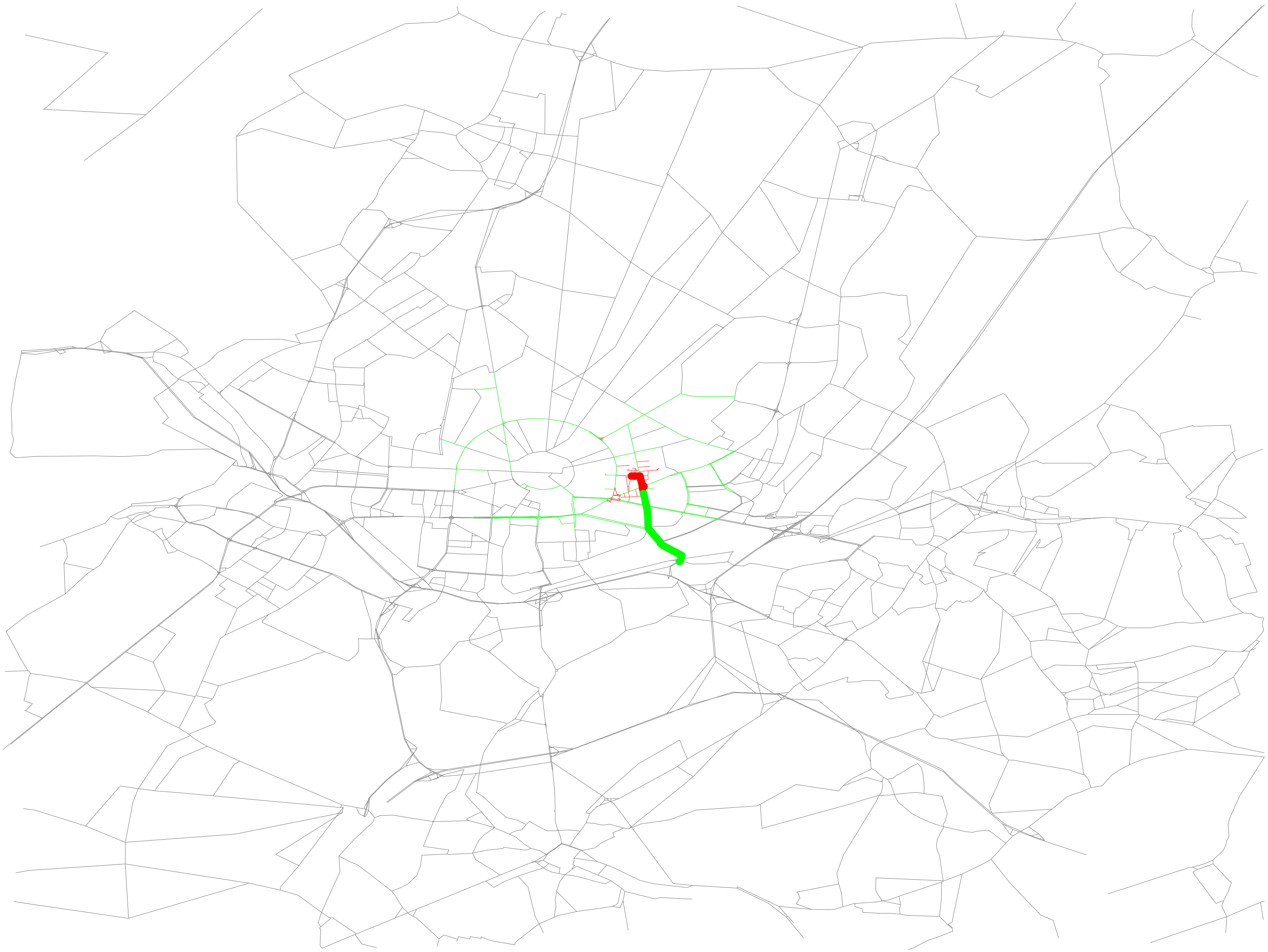
Query

Example: from Karlsruhe, Am Fasanengarten 5

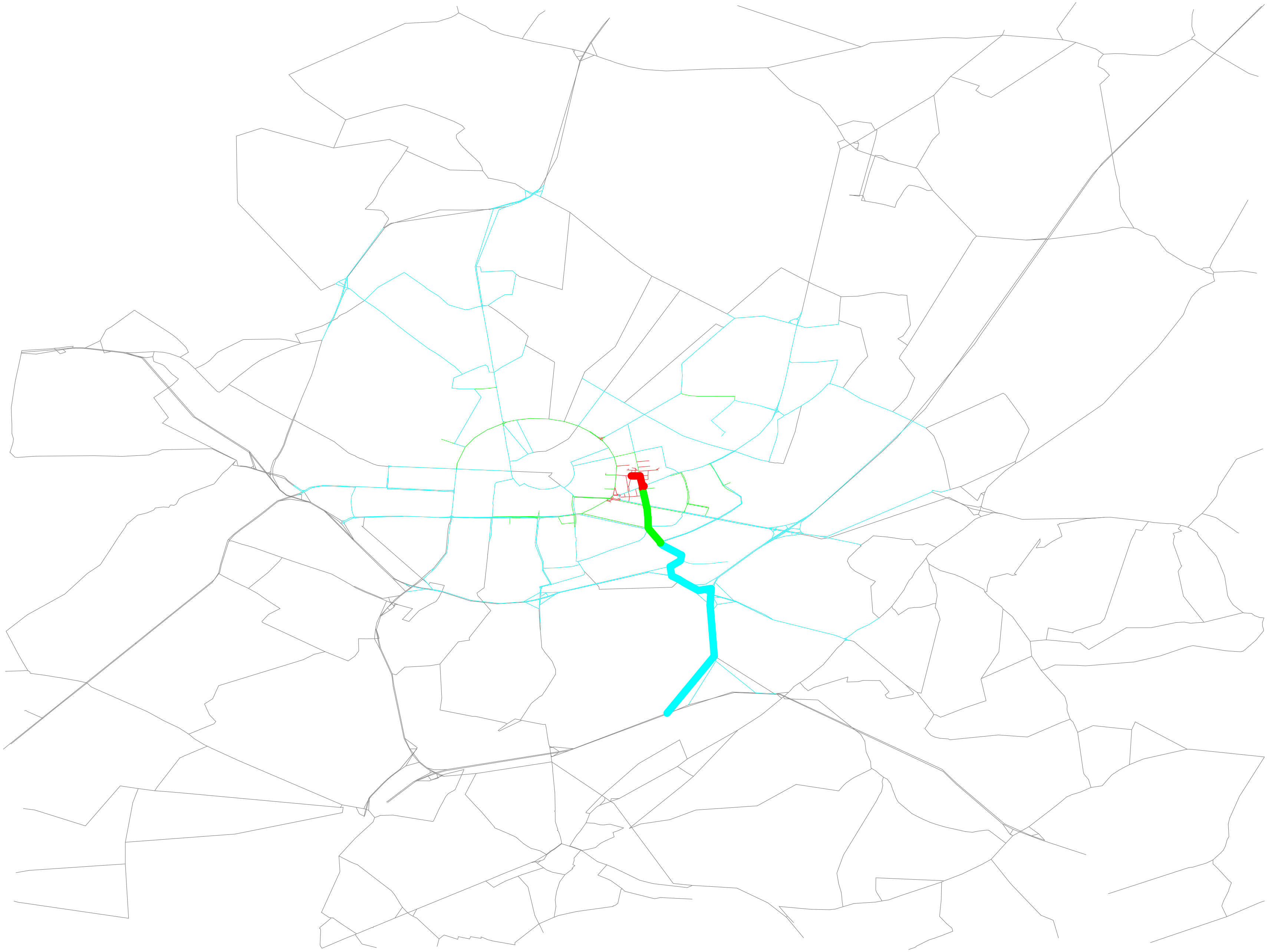
to Palma de Mallorca

Bounding Box: 20 km Level 0 Search Space

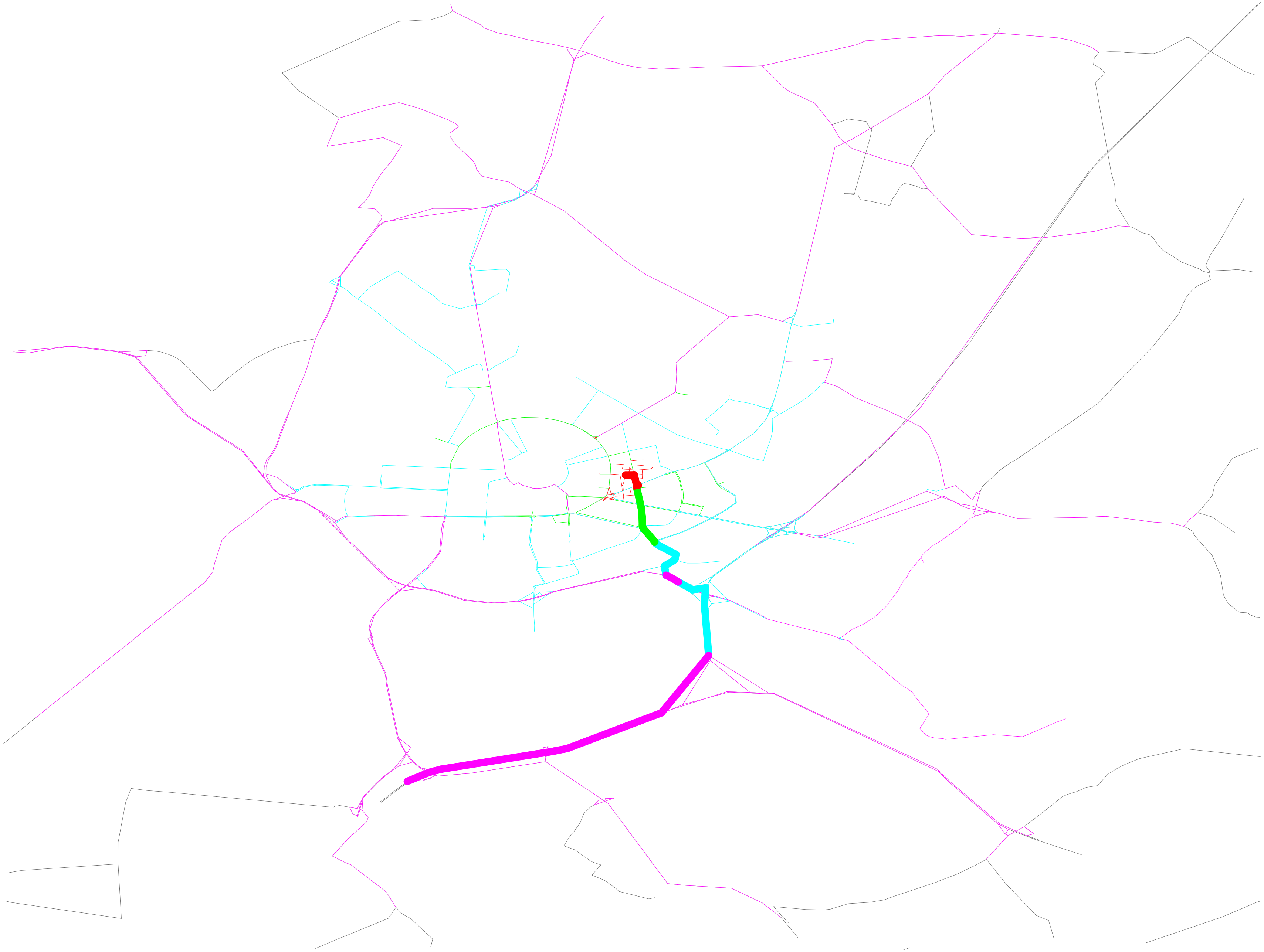
Bounding Box: 20 km Level 1 Search Space



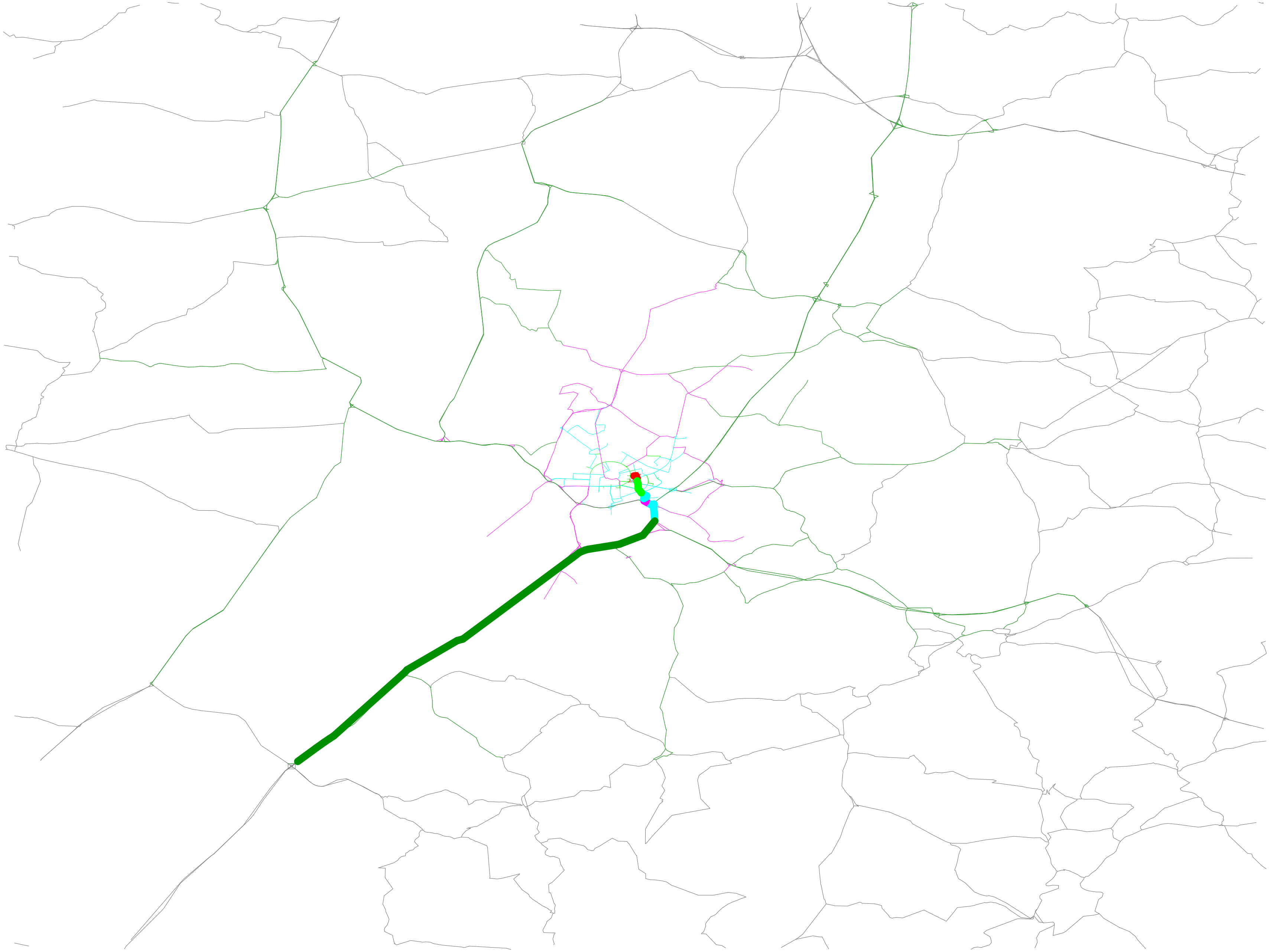
Bounding Box: 20 km Level 2 Search Space



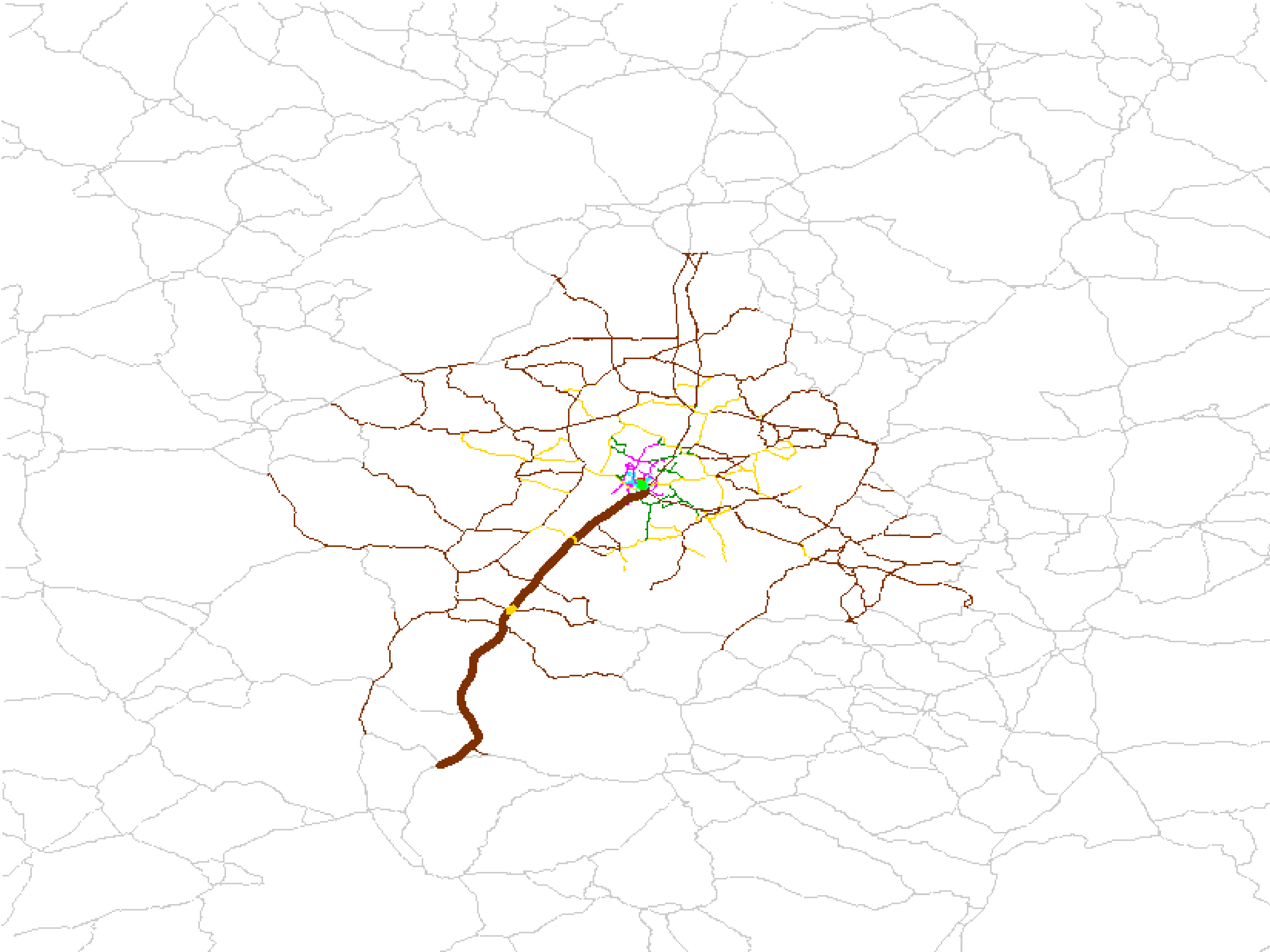
Bounding Box: 20 km Level 3 Search Space



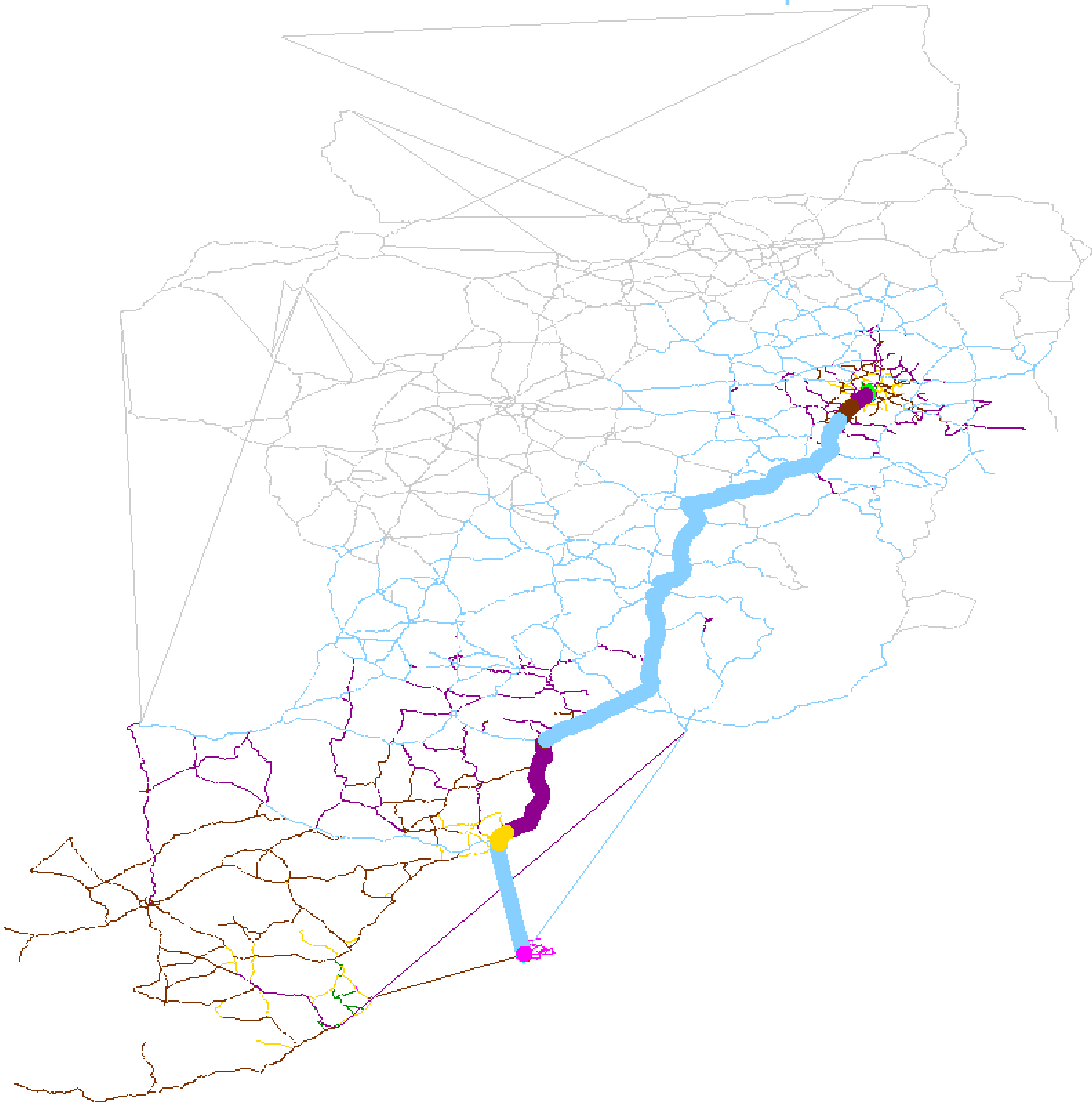
Bounding Box: 80 km Level 4 Search Space



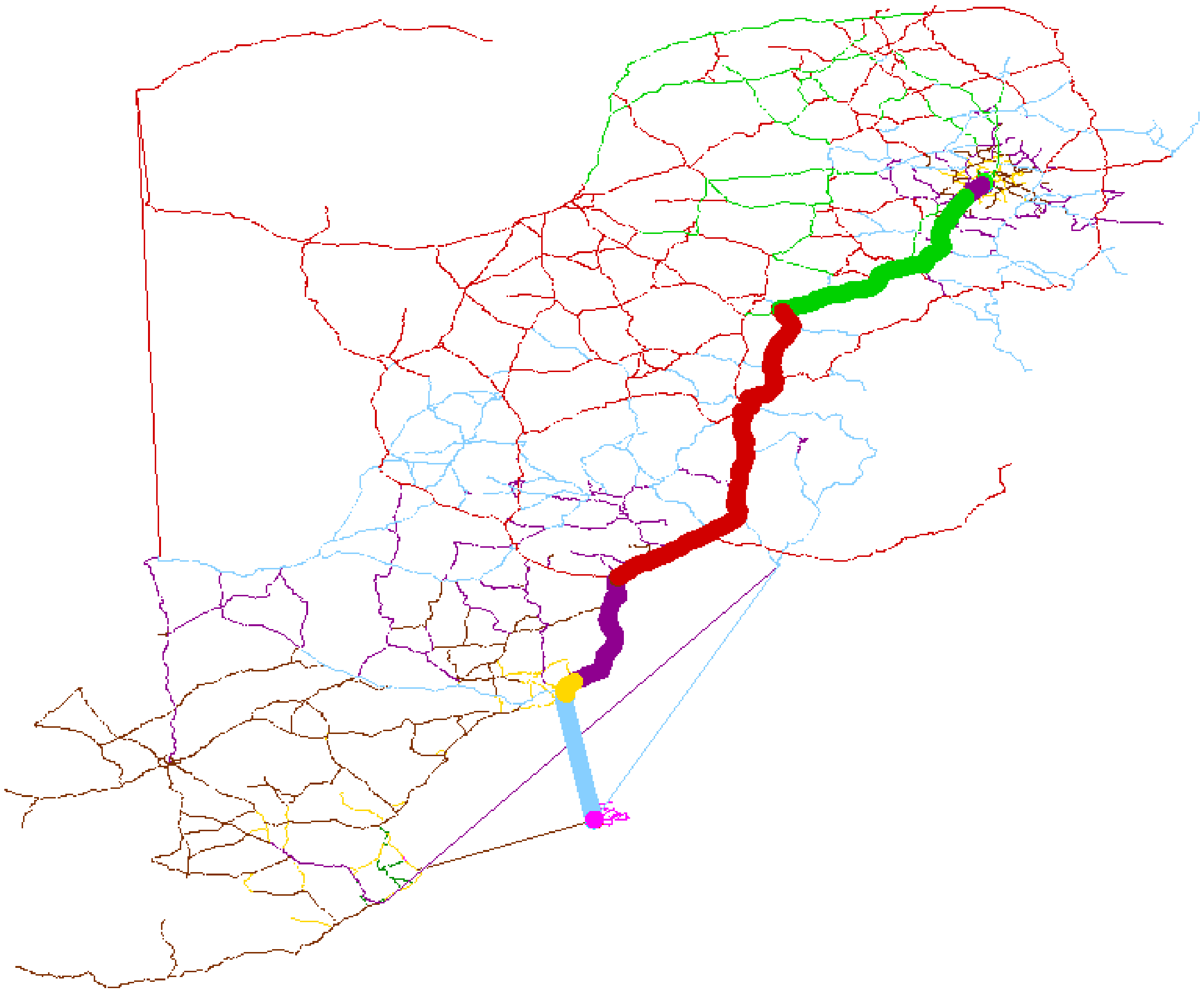
Bounding Box: 400 km Level 6 Search Space



Level 8 Search Space



Level 10 Search Space

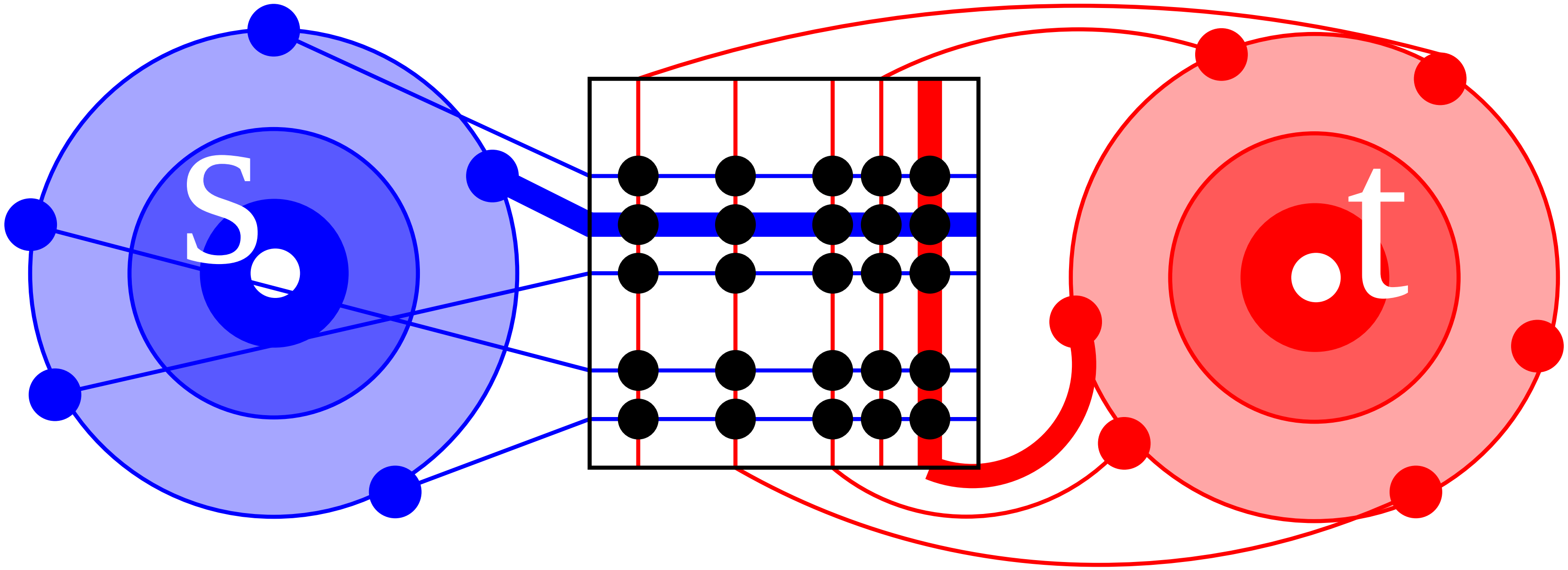


Optimisation: Distance Table

Construction:

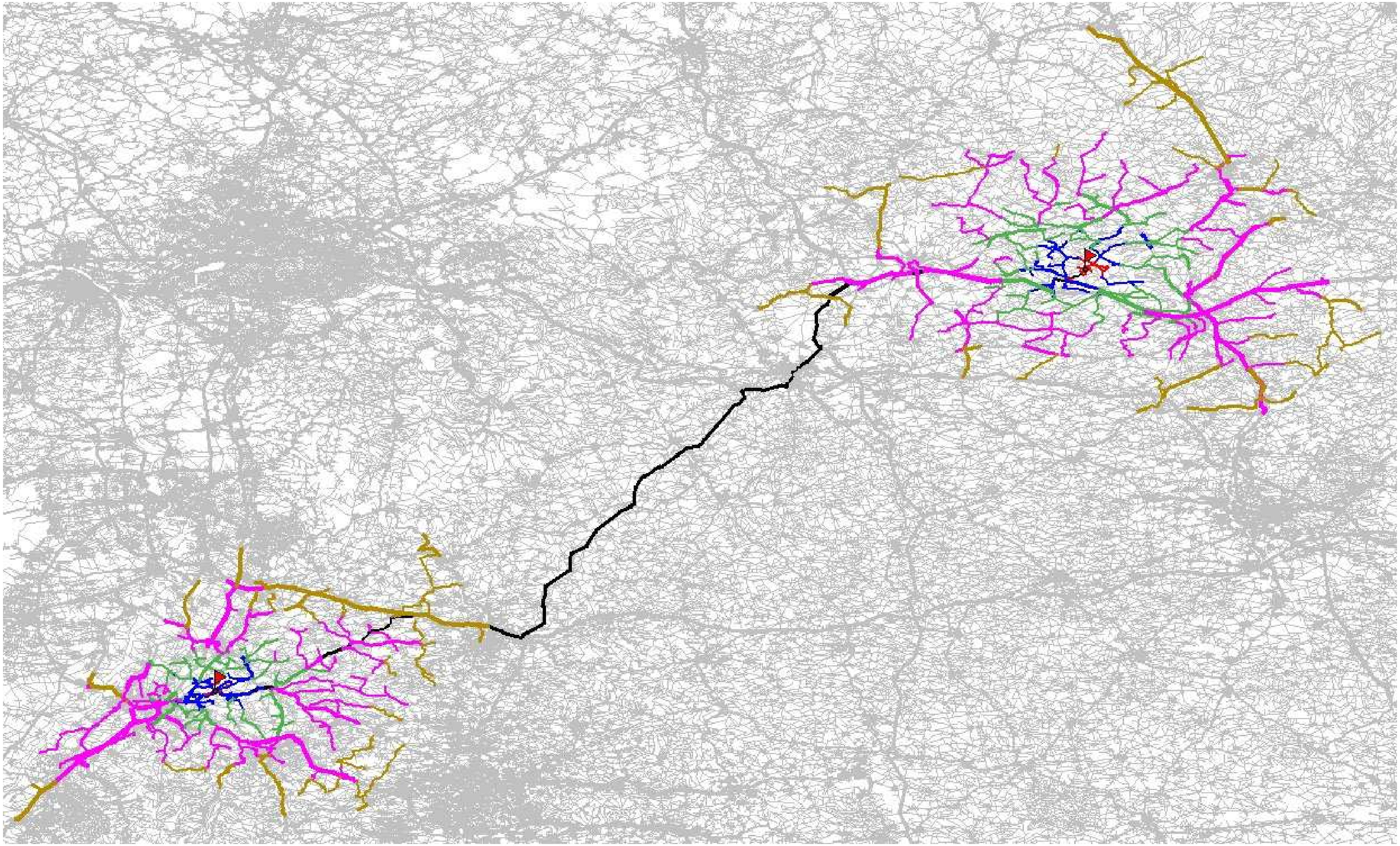
- Construct fewer levels. e.g. 4 instead of 9
- Compute an all-pairs distance table for the topmost level L. $13\,465 \times 13\,465$ entries

Distance Table Query:

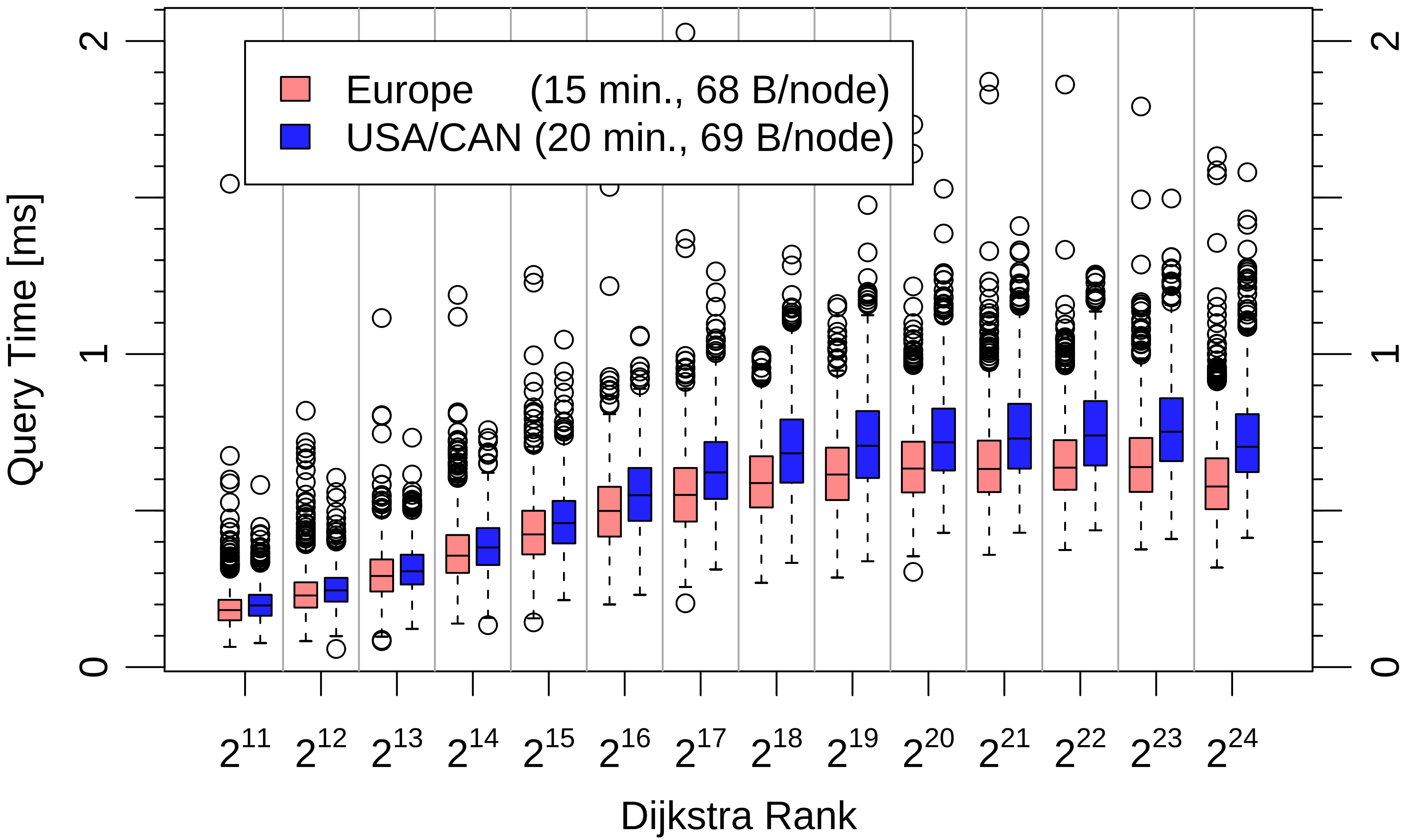


- Abort the search when all entrance points in the core of level L have been encountered. $\approx$ 55 for each direction
- Use the distance table to bridge the gap. $\approx 55 \times 55$ entries

Distance Table: Search Space Example

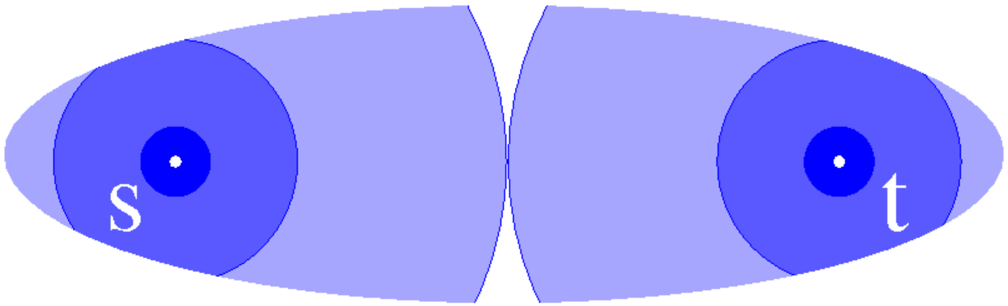


Local Queries (Highway Hierarchies)



Combination Goal Directed Search (landmarks)

[with D. Delling, D. Wagner]



- About 20 % faster than HHs + distance tables
- Significant speedup for approximate queries

Many-to-Many Routing

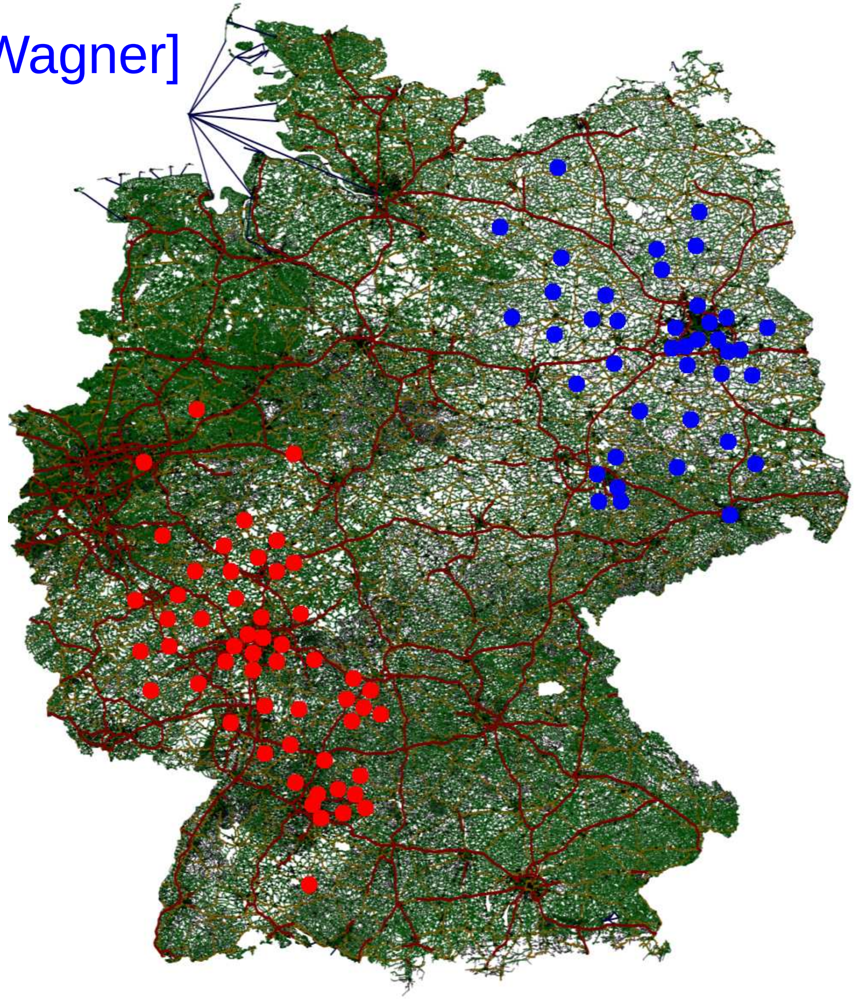
[with S. Knopp, F. Schulz (PTV AG), D. Wagner]

Find distances for all $(s,t) \in S \times T$

Applications: vehicle routing, TSP,

traffic simulation,

subroutine in peprocessing algorithms.



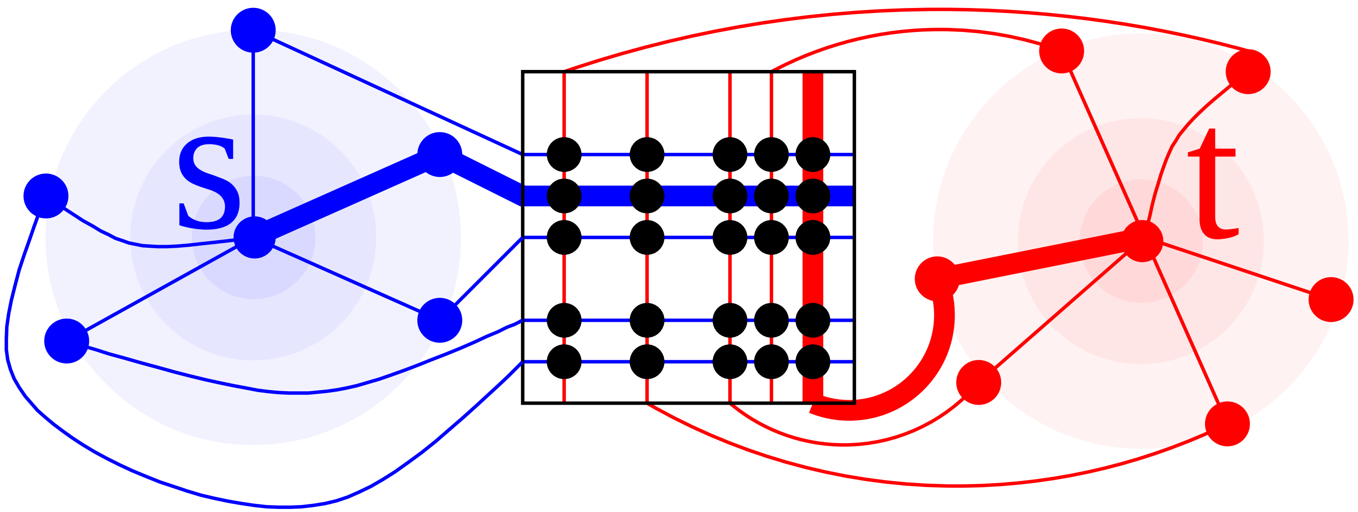
For example,

$10\,000 \times 10\,000$ table

in $\approx$1 min

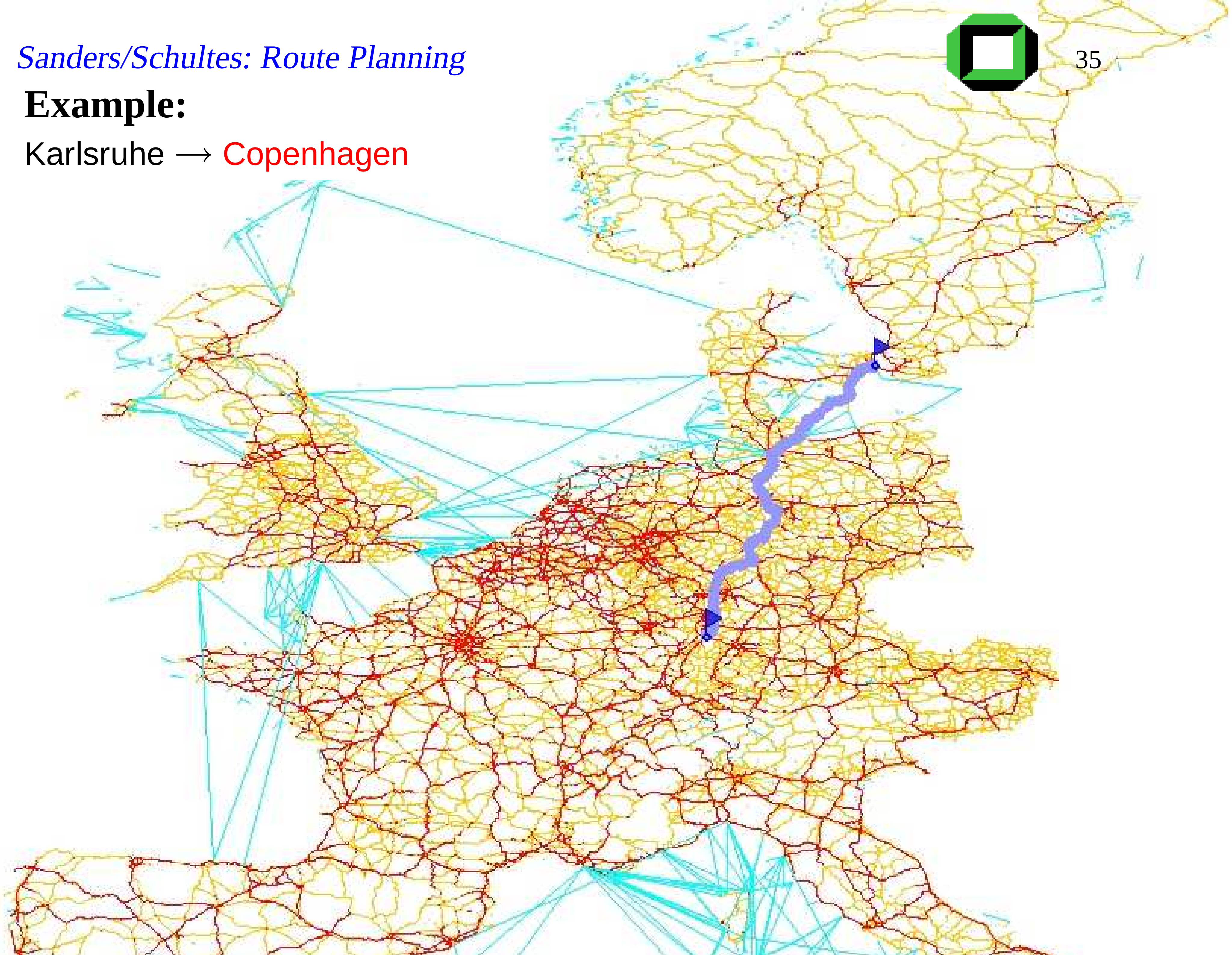
Transit-Node Routing

[with H. Bast and S. Funke, DIMACS 06, Alenex 07, **Science 07**]



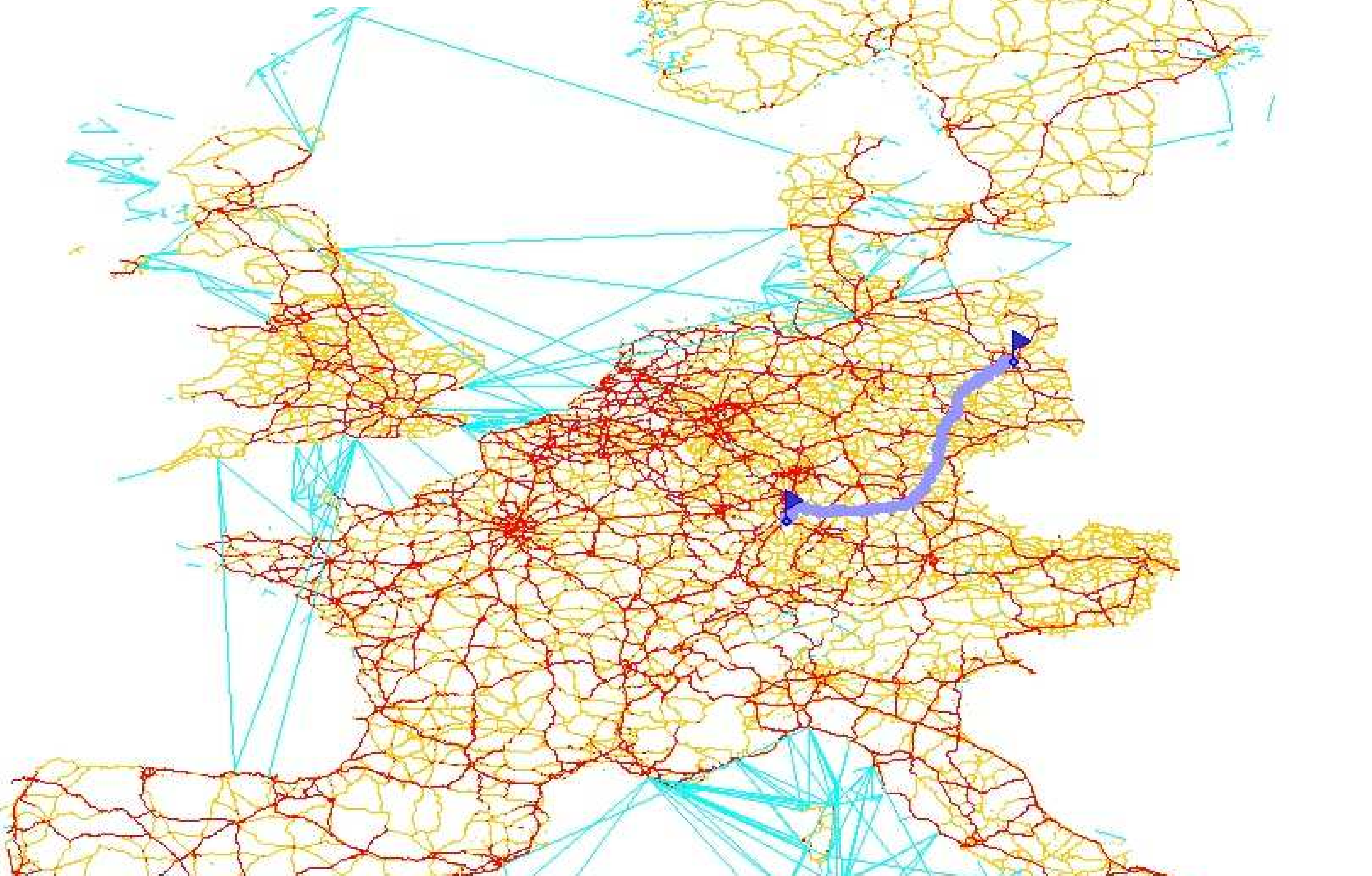
Example:

Karlsruhe $\longrightarrow$ Copenhagen



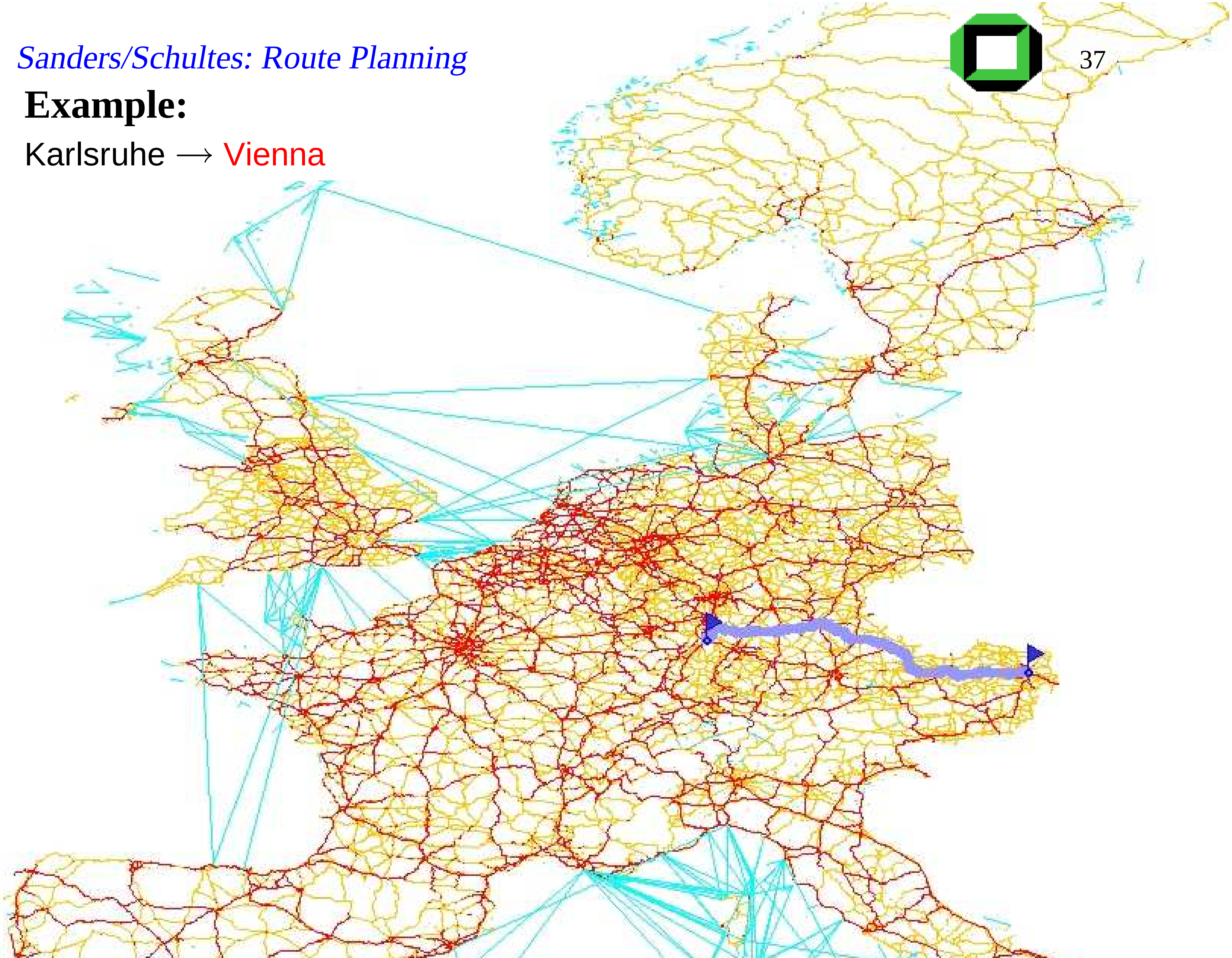
Example:

Karlsruhe ⟶ Berlin



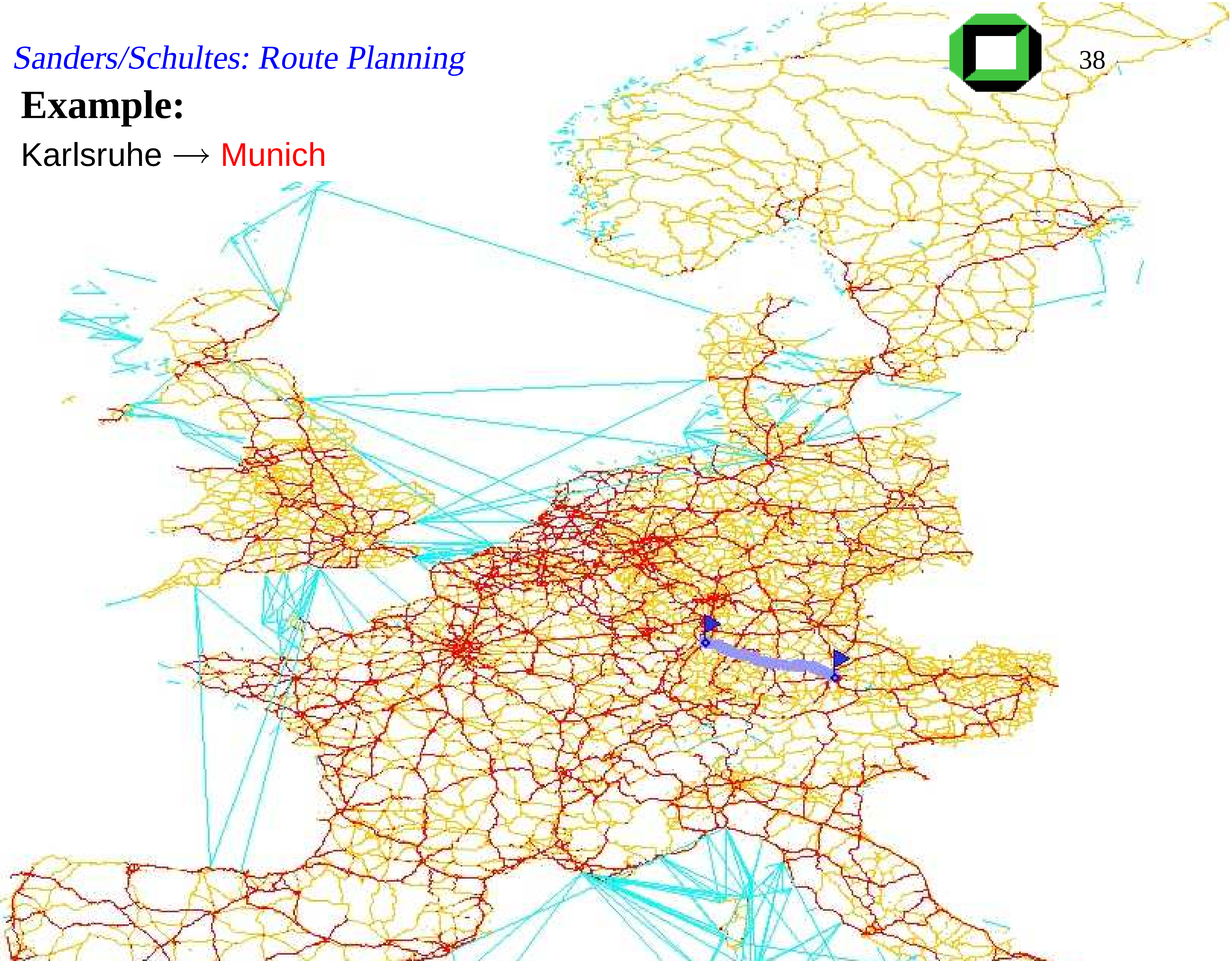
Example:

Karlsruhe → Vienna



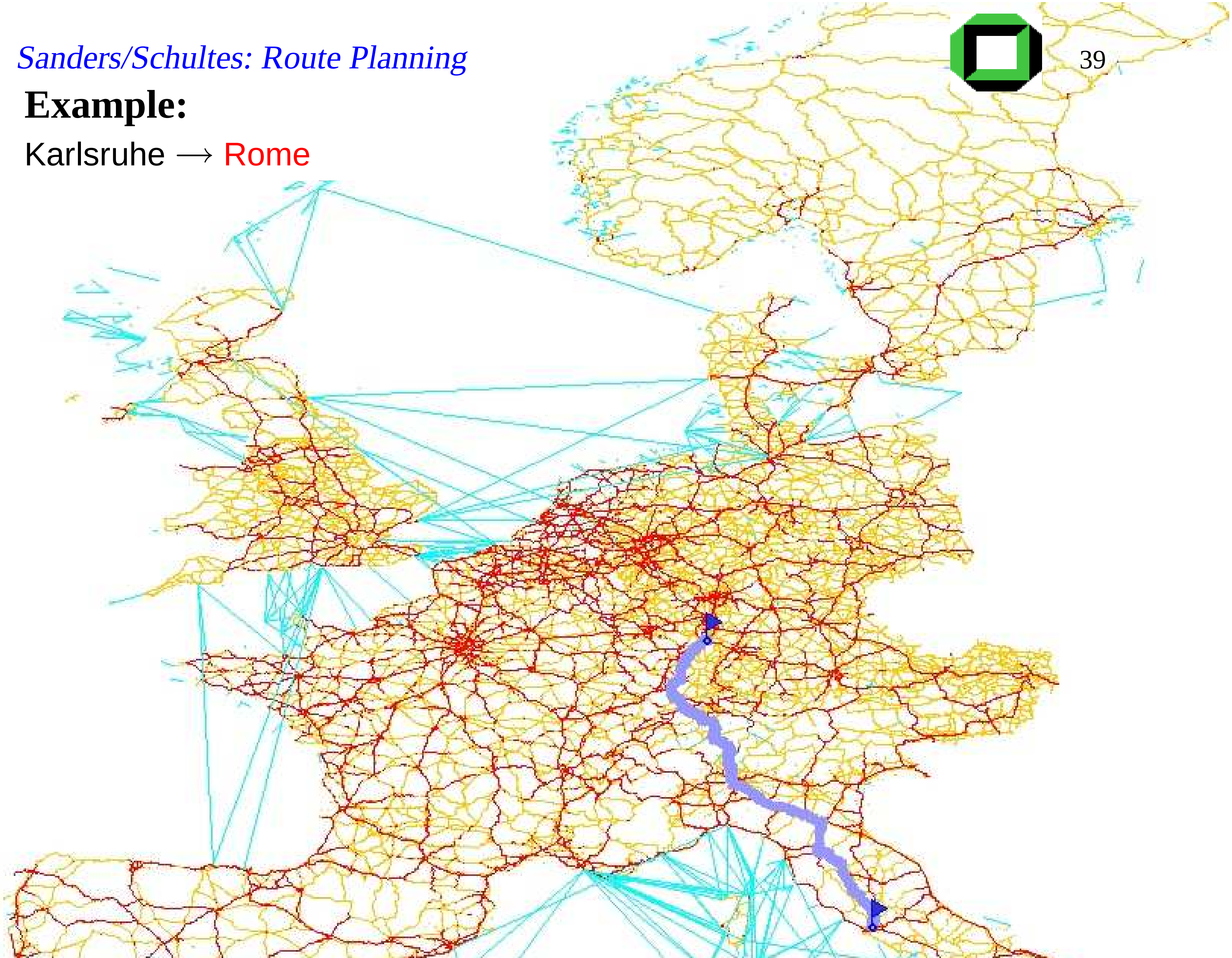
Example:

Karlsruhe $\longrightarrow$ Munich



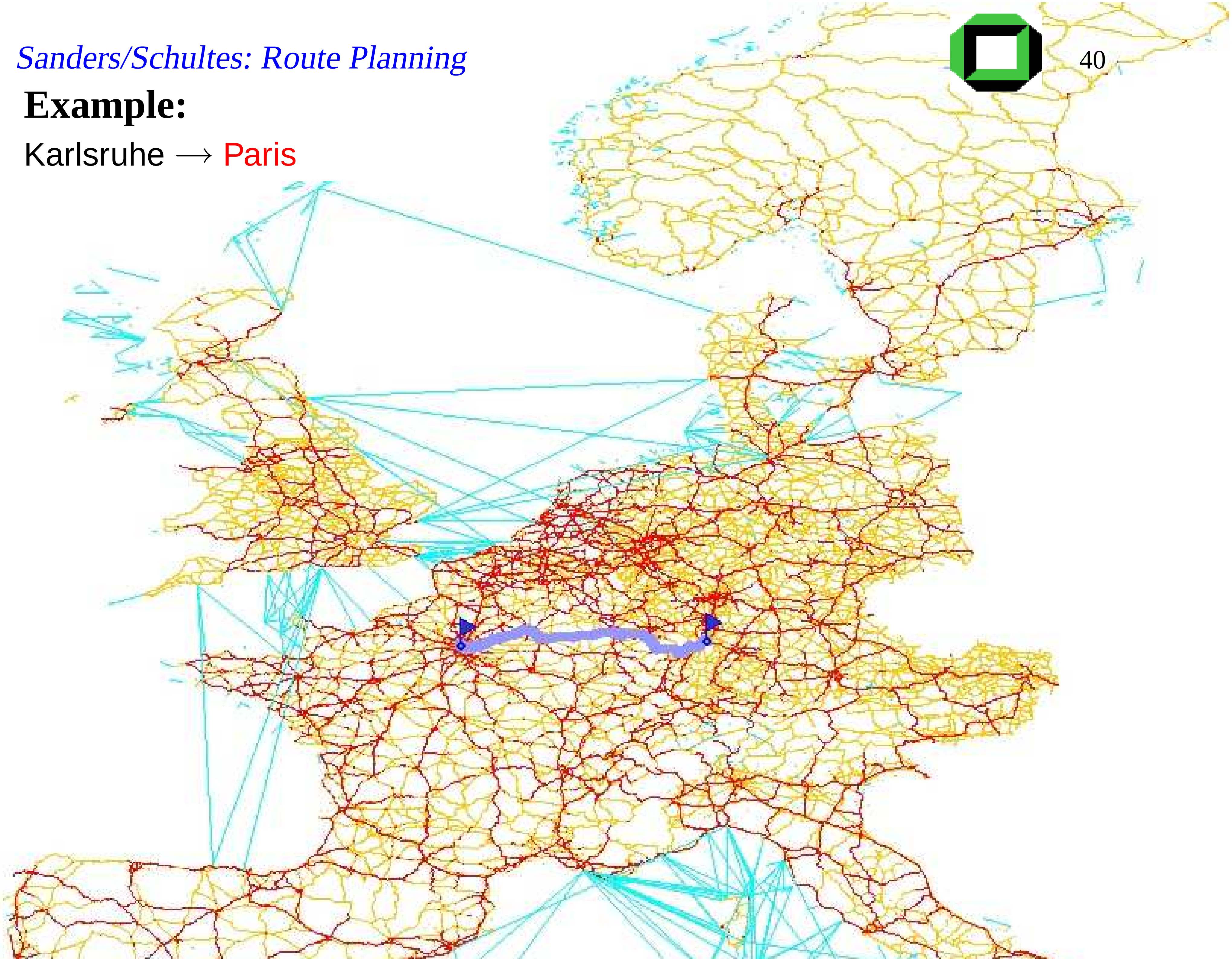
Example:

Karlsruhe → Rome



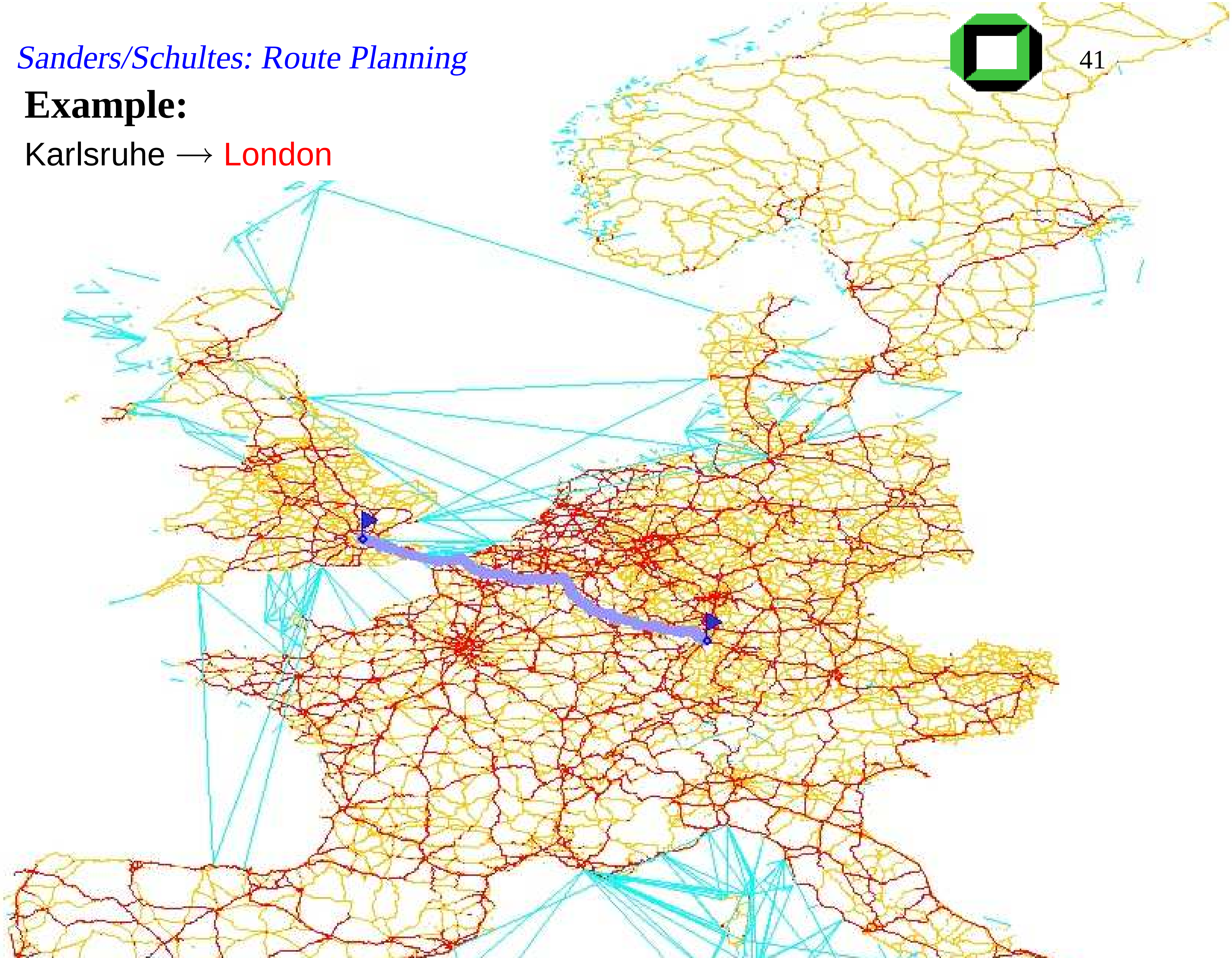
Example:

Karlsruhe ⟶ Paris



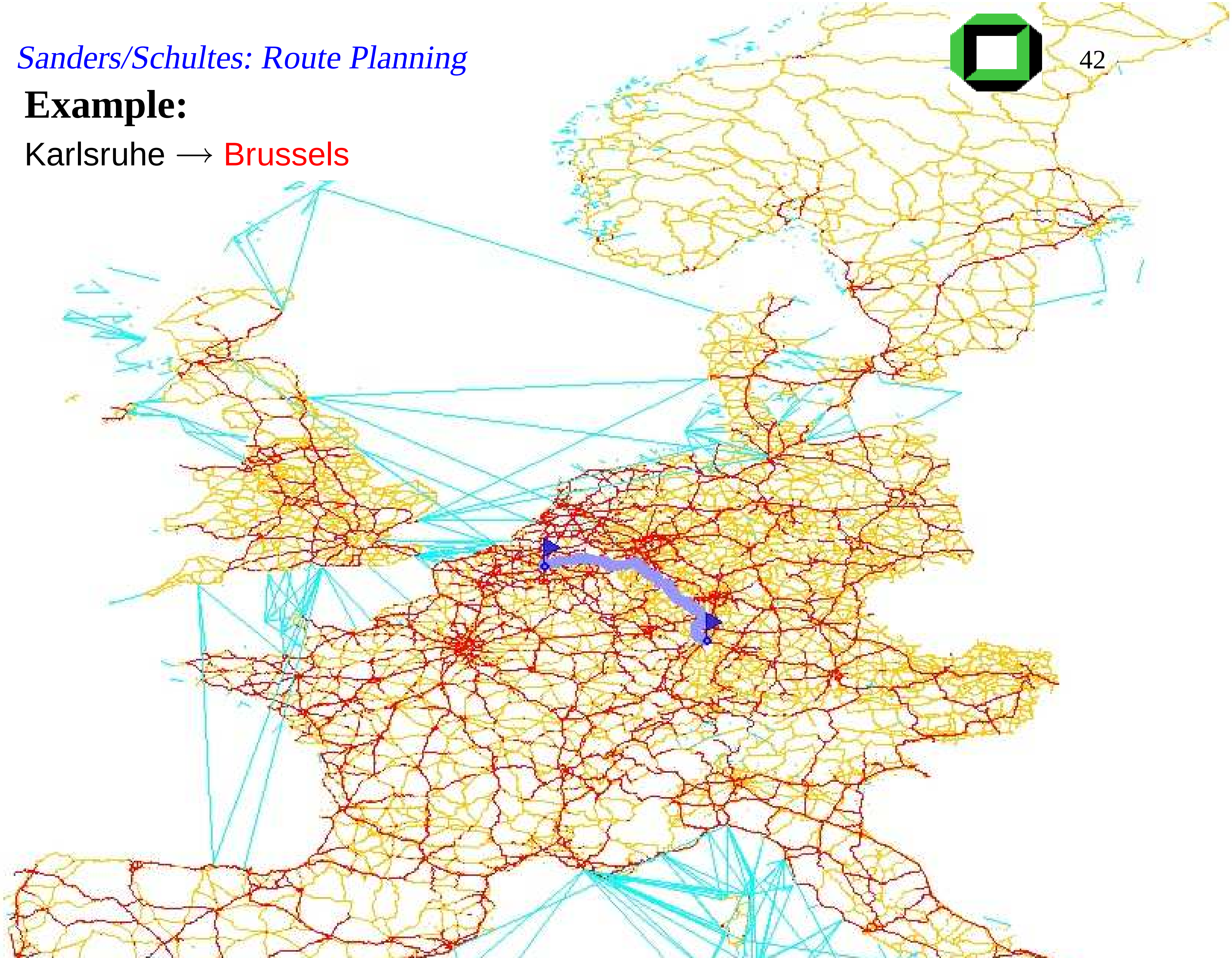
Example:

Karlsruhe $\longrightarrow$ London



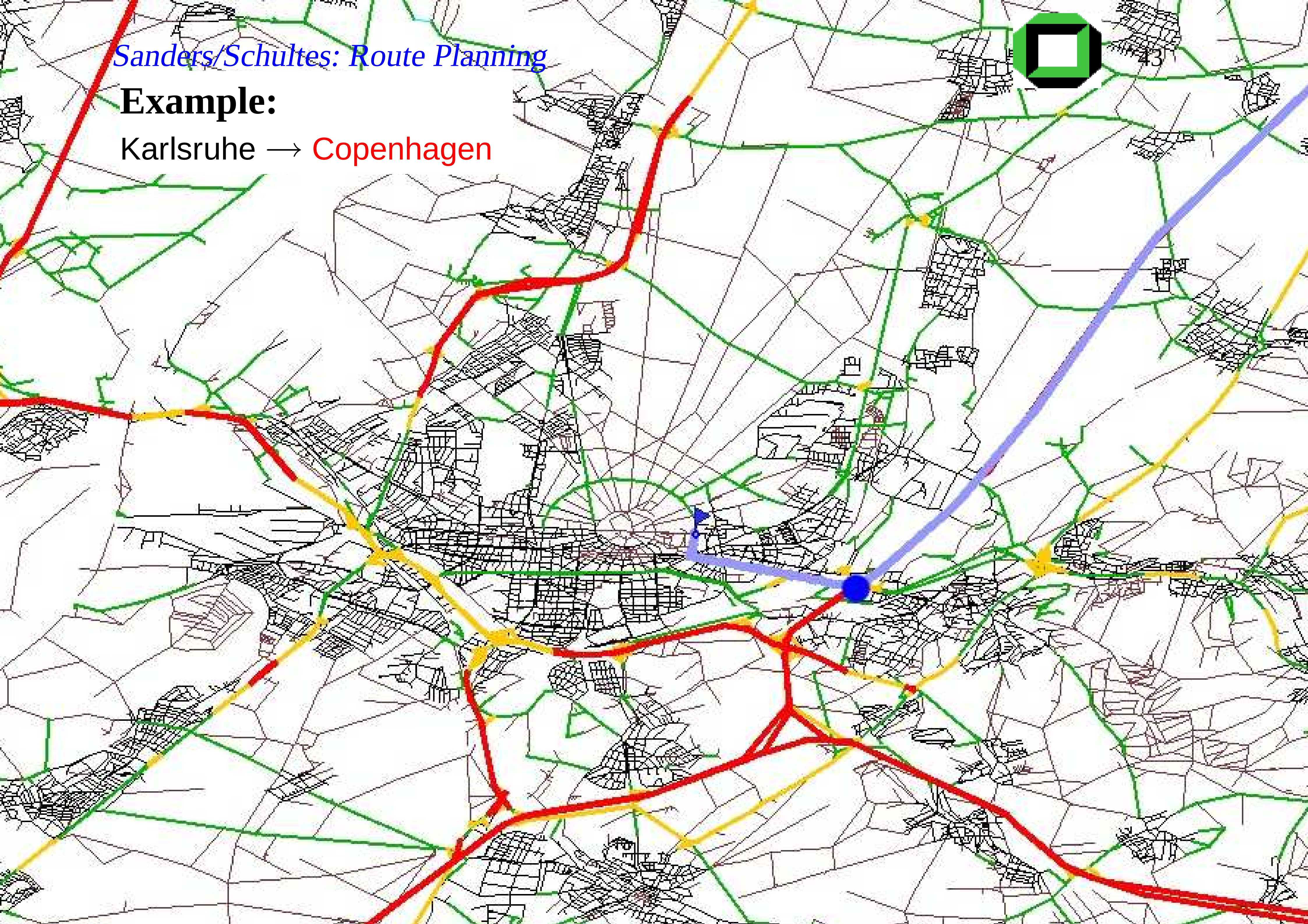
Example:

Karlsruhe → Brussels



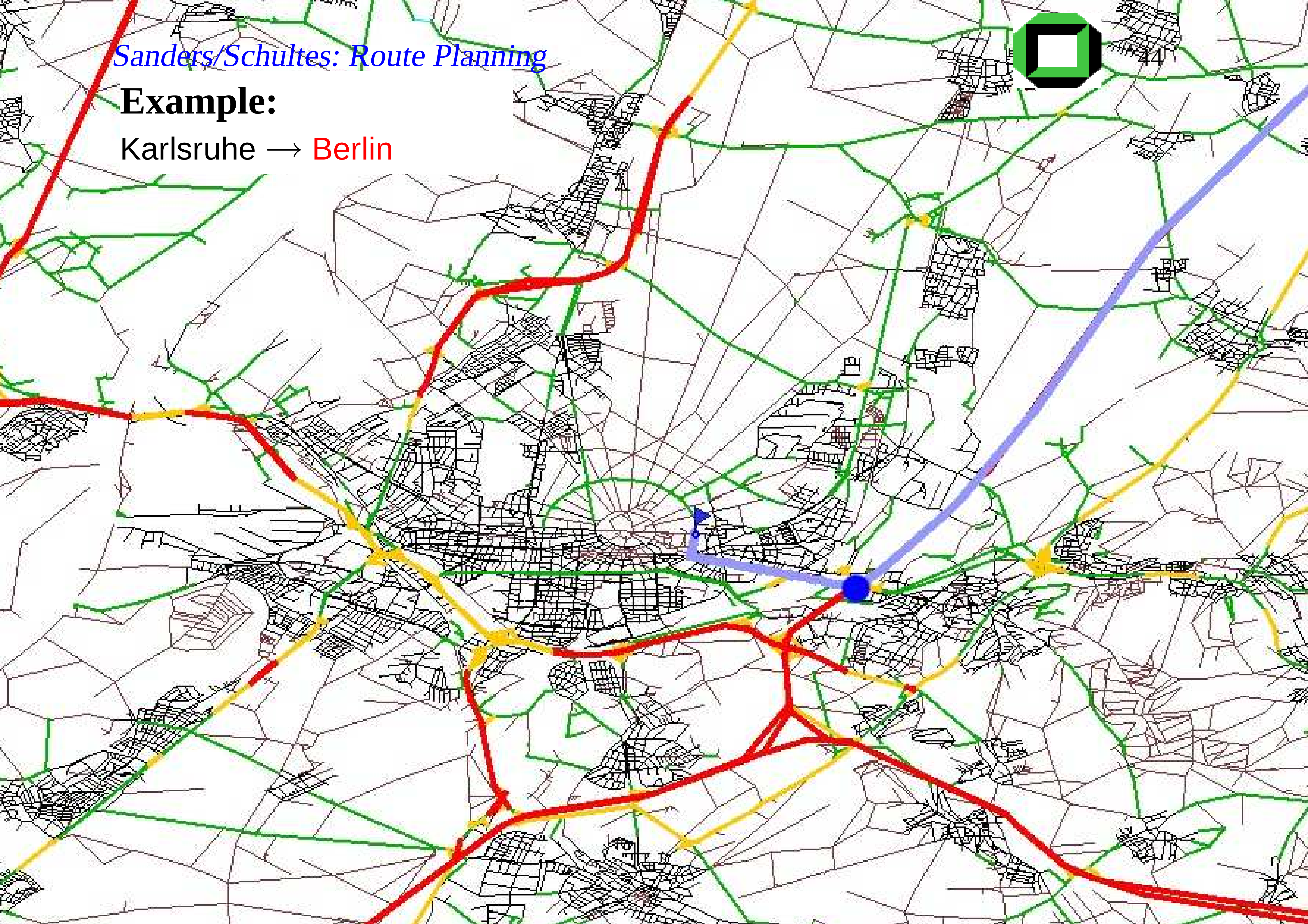
Example:

Karlsruhe → Copenhagen



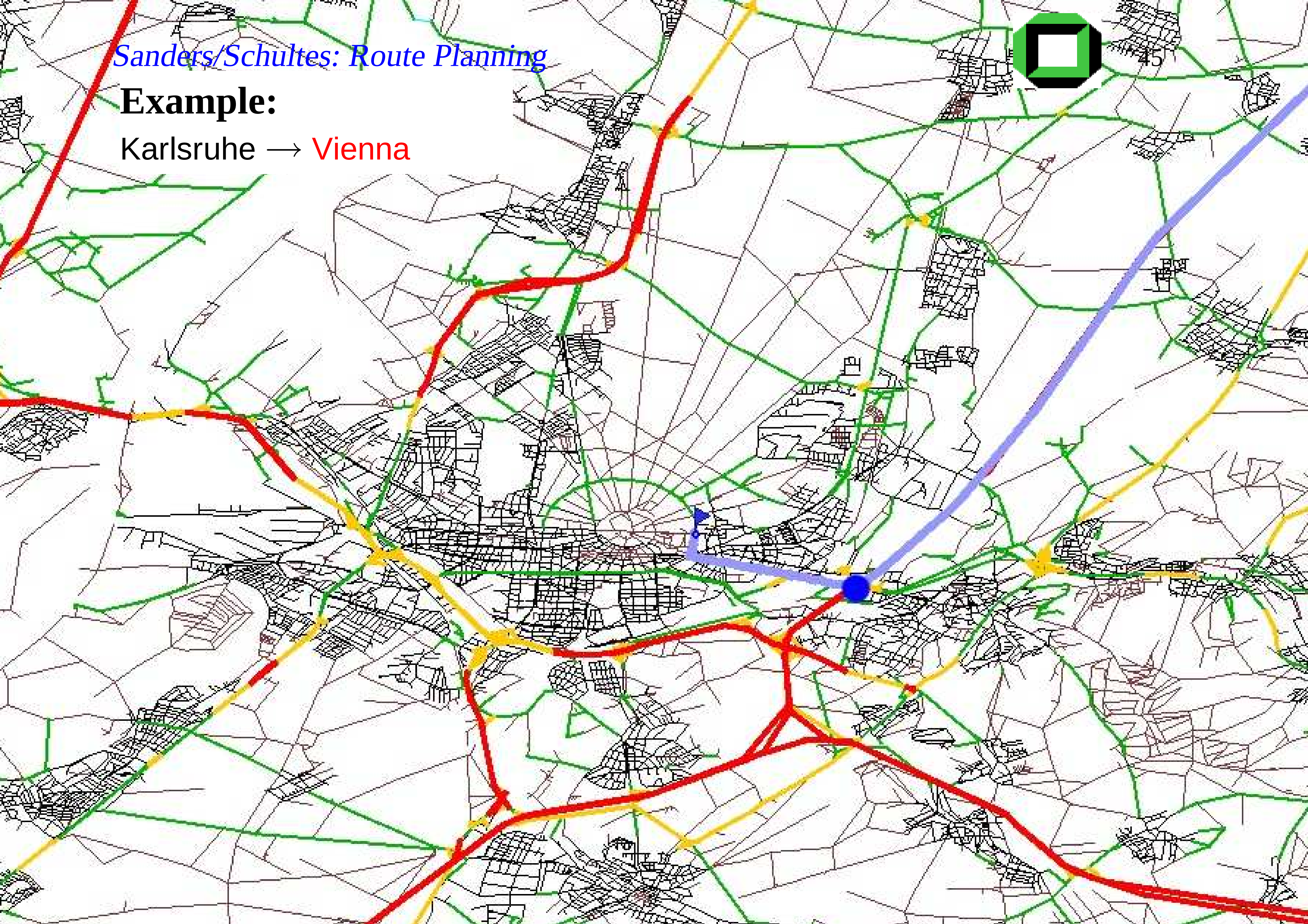
Example:

Karlsruhe → Berlin



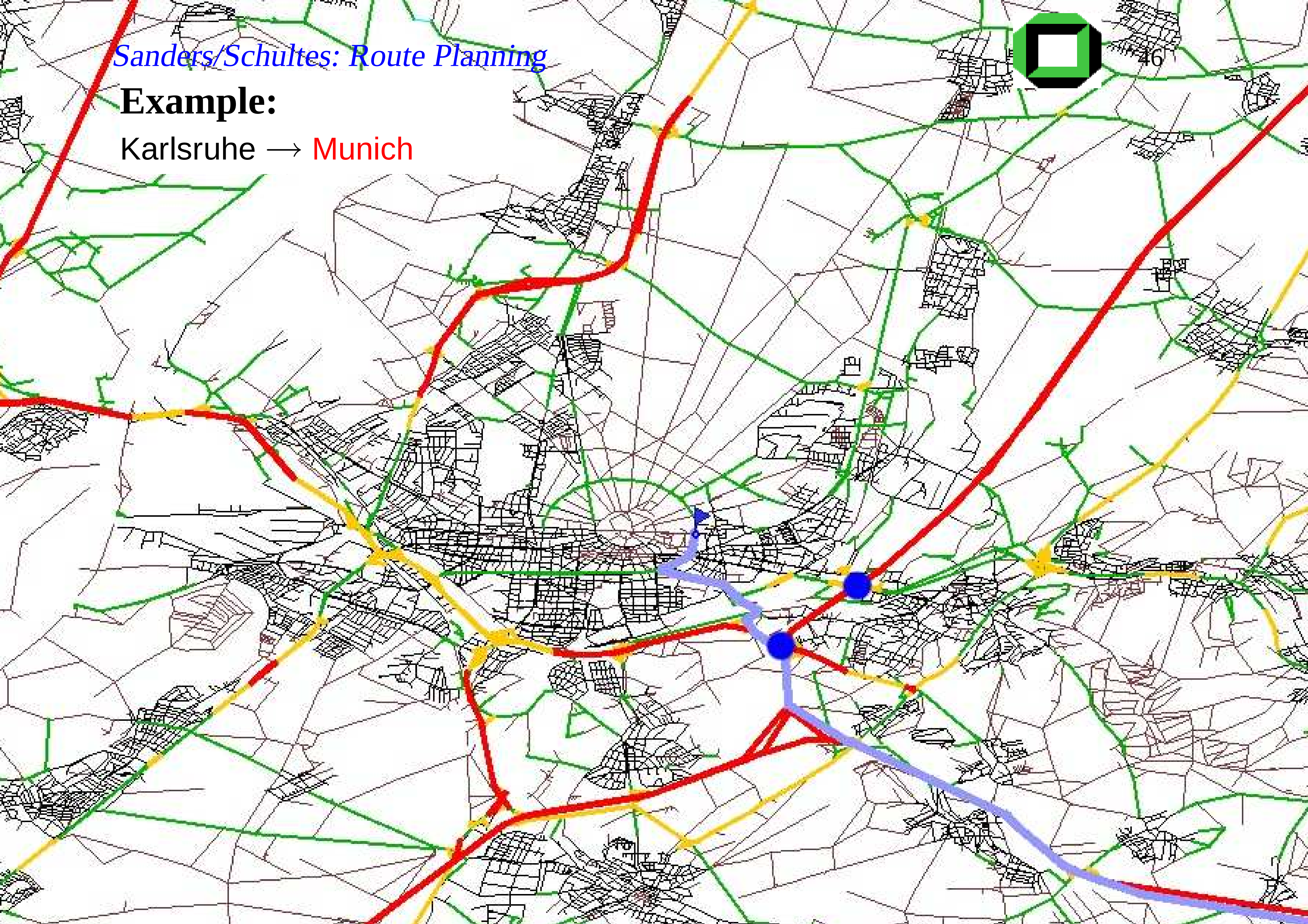
Example:

Karlsruhe → Vienna



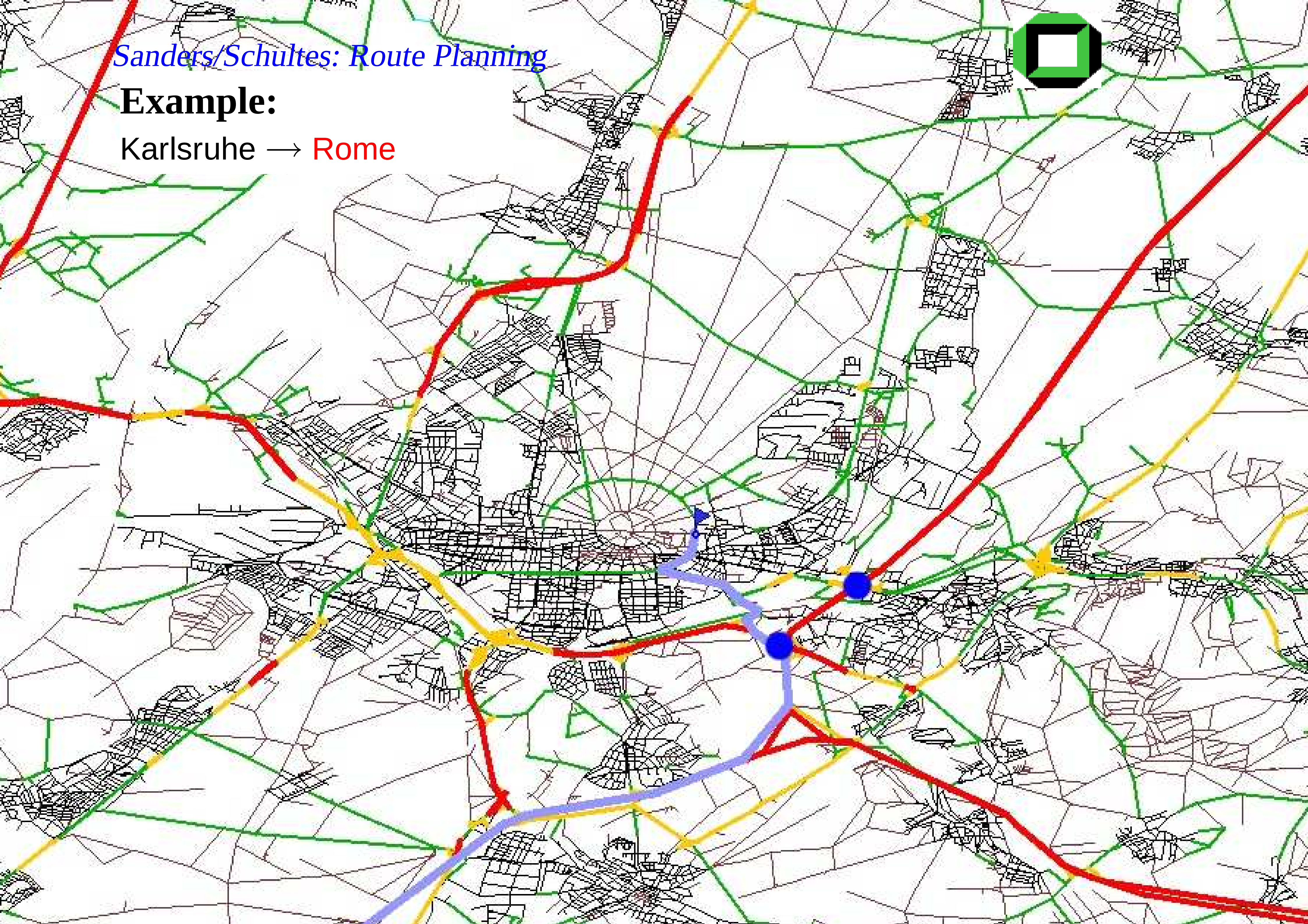
Example:

Karlsruhe → Munich



Example:

Karlsruhe → Rome



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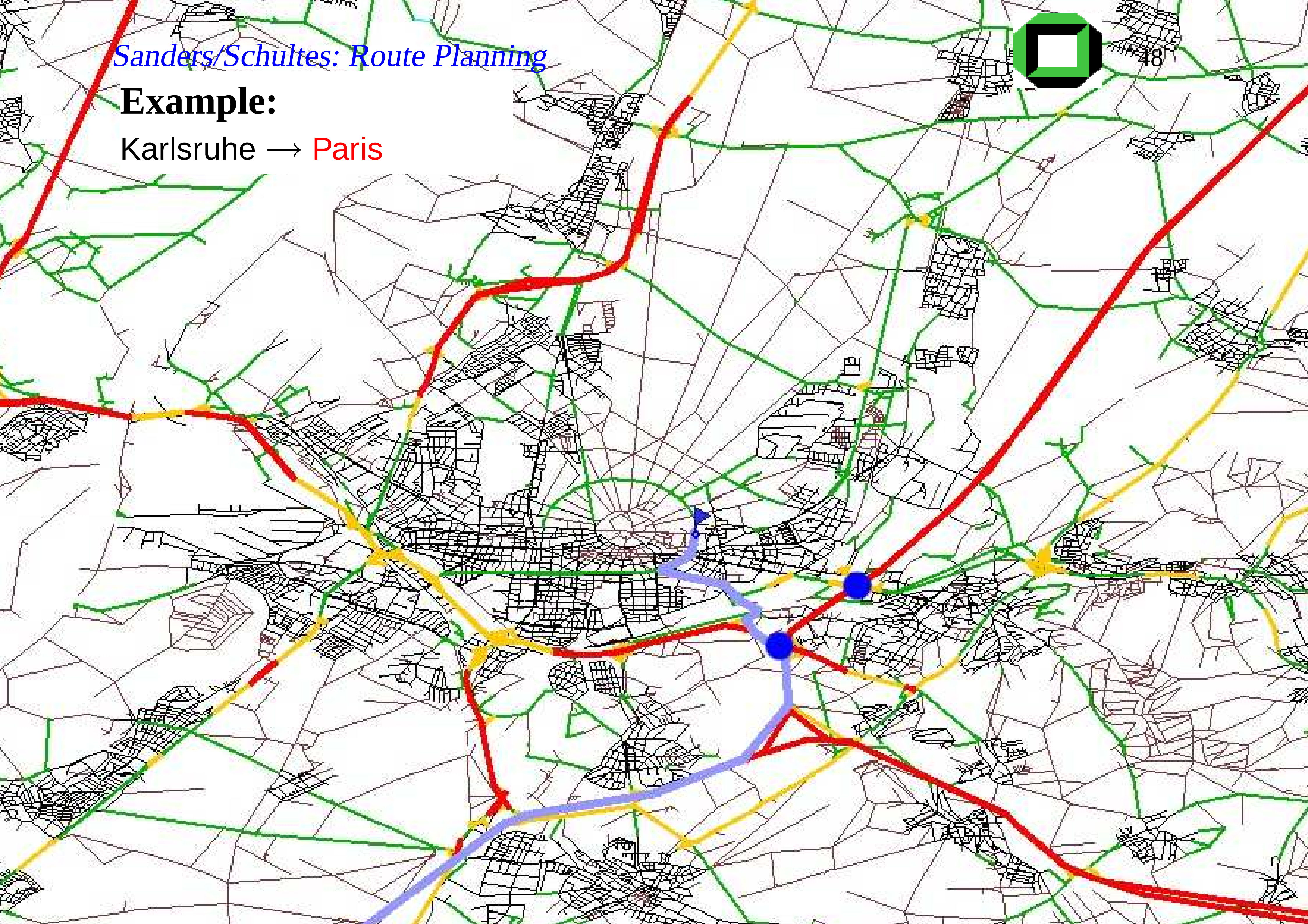
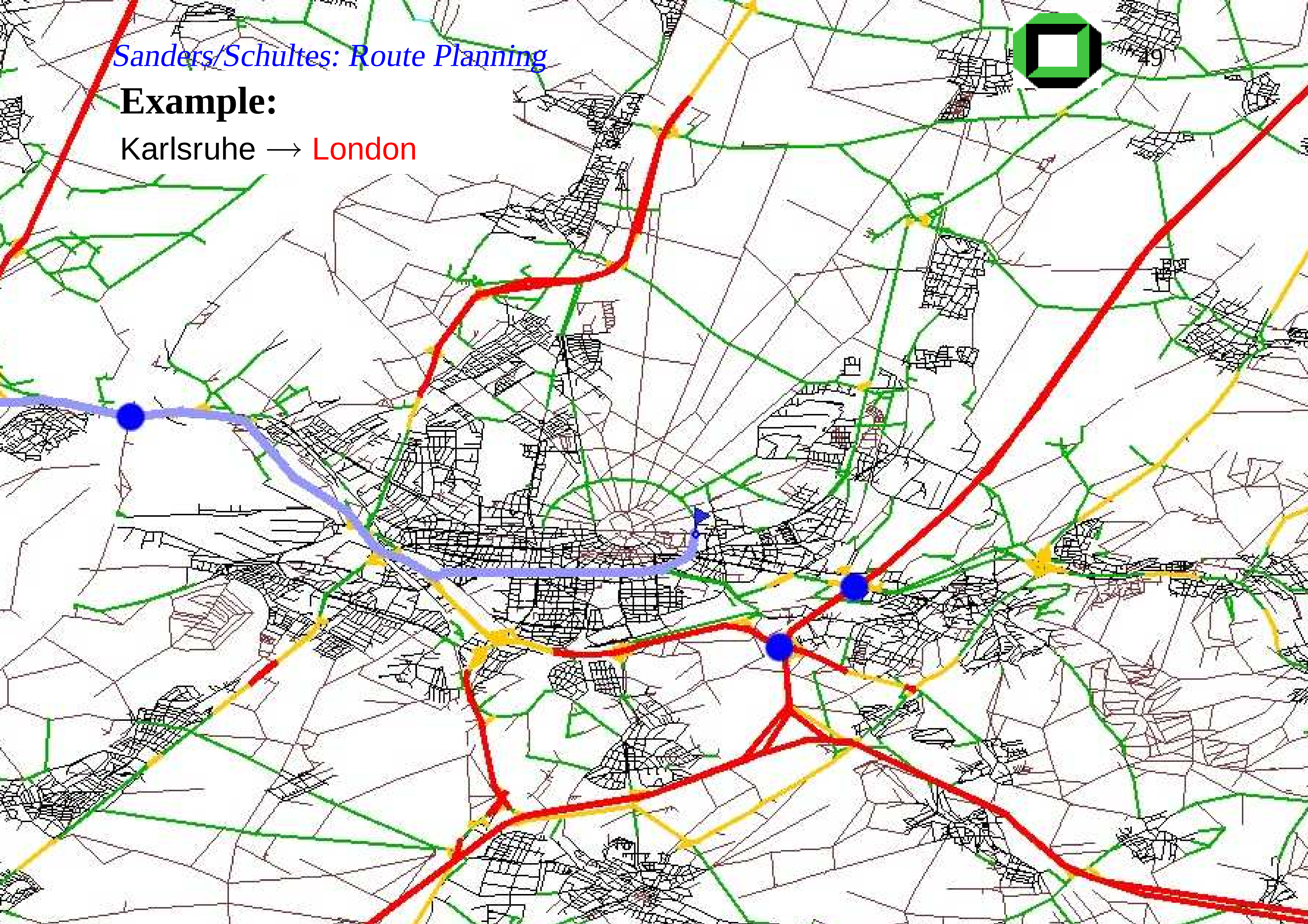
Karlsruhe → Paris



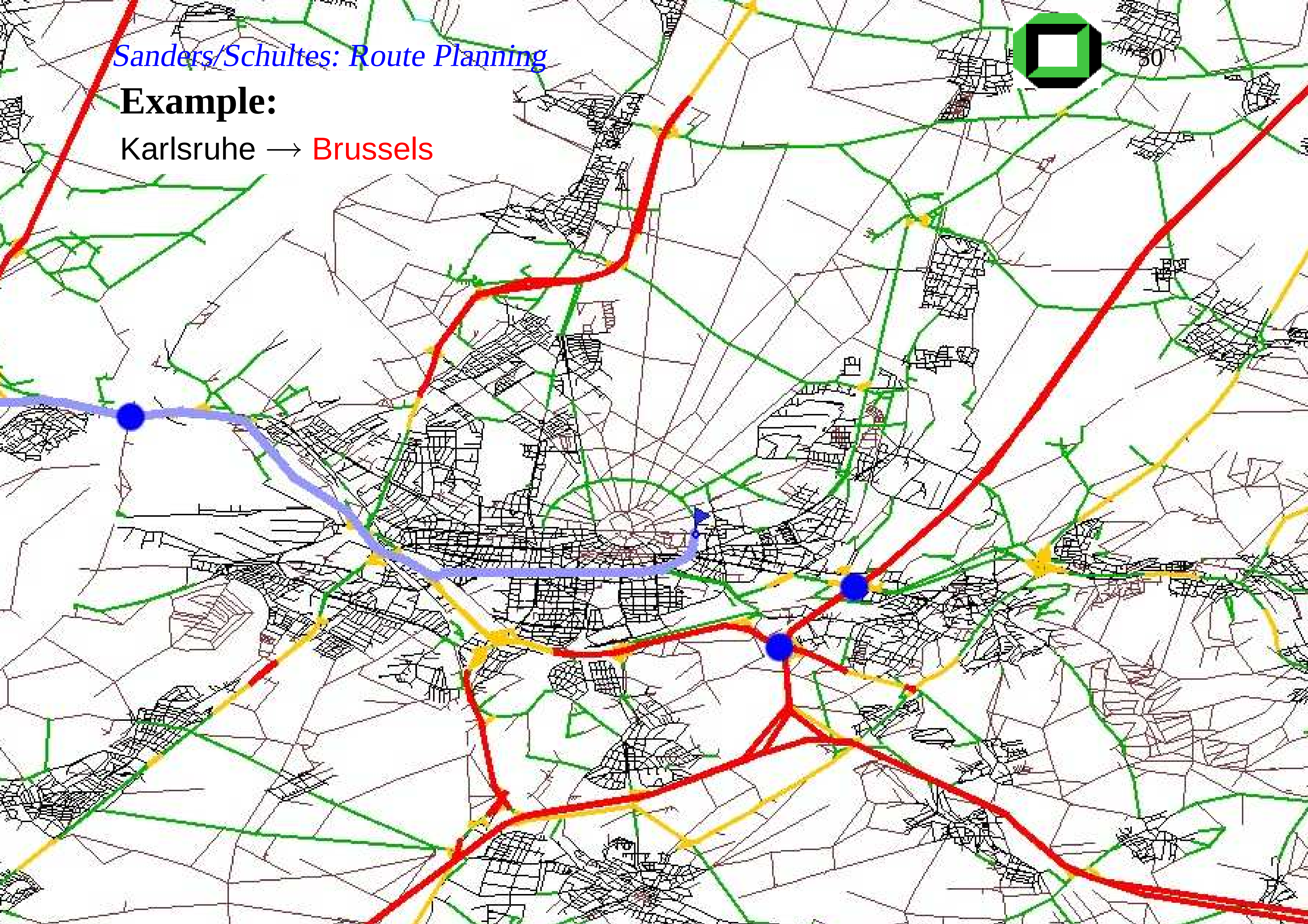
Example:

Karlsruhe → London



Example:

Karlsruhe → Brussels



Observations for long-distance travel

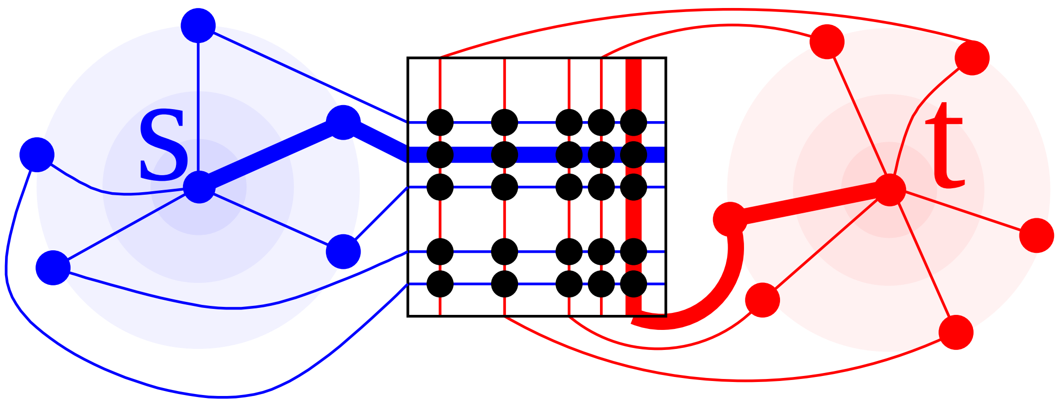
Europe ≈

1. leaves area via one of only a few **access points** 10

   ⇝ store them for each node

2. all access points come from a small set of **transit nodes** 10 000

   ⇝ store distances between all transit-node pairs



Transit-Node Routing

Preprocessing: Our Implementation

- identify transit-node set $\mathcal{T} \subseteq V$ — upper levels of HH
- compute complete $|\mathcal{T}| \times |\mathcal{T}|$ distance table — many-to-many
- for each node: identify its access points (mapping $A : V \longrightarrow 2^{\mathcal{T}}$), store the distances — HH-search

Query (source s and target t given): compute

$$d_{\mathrm{top}}(s,t) := \min\{d(s,u)+d(u,v)+d(v,t) : u \in A(s), v \in A(t)\}$$

Transit-Node Routing

Our Implementation

Locality Filter:

local cases must be filtered ($\leadsto$ special treatment)

$L : V \times V \rightarrow \{\mathsf{true}, \mathsf{false}\}$

$\neg L(s,t)$ implies $d(s,t) = d_{\text{top}}(s,t)$

intersection of disks around s and t

Additional Layers:

Local cases: use secondary transit-node set.

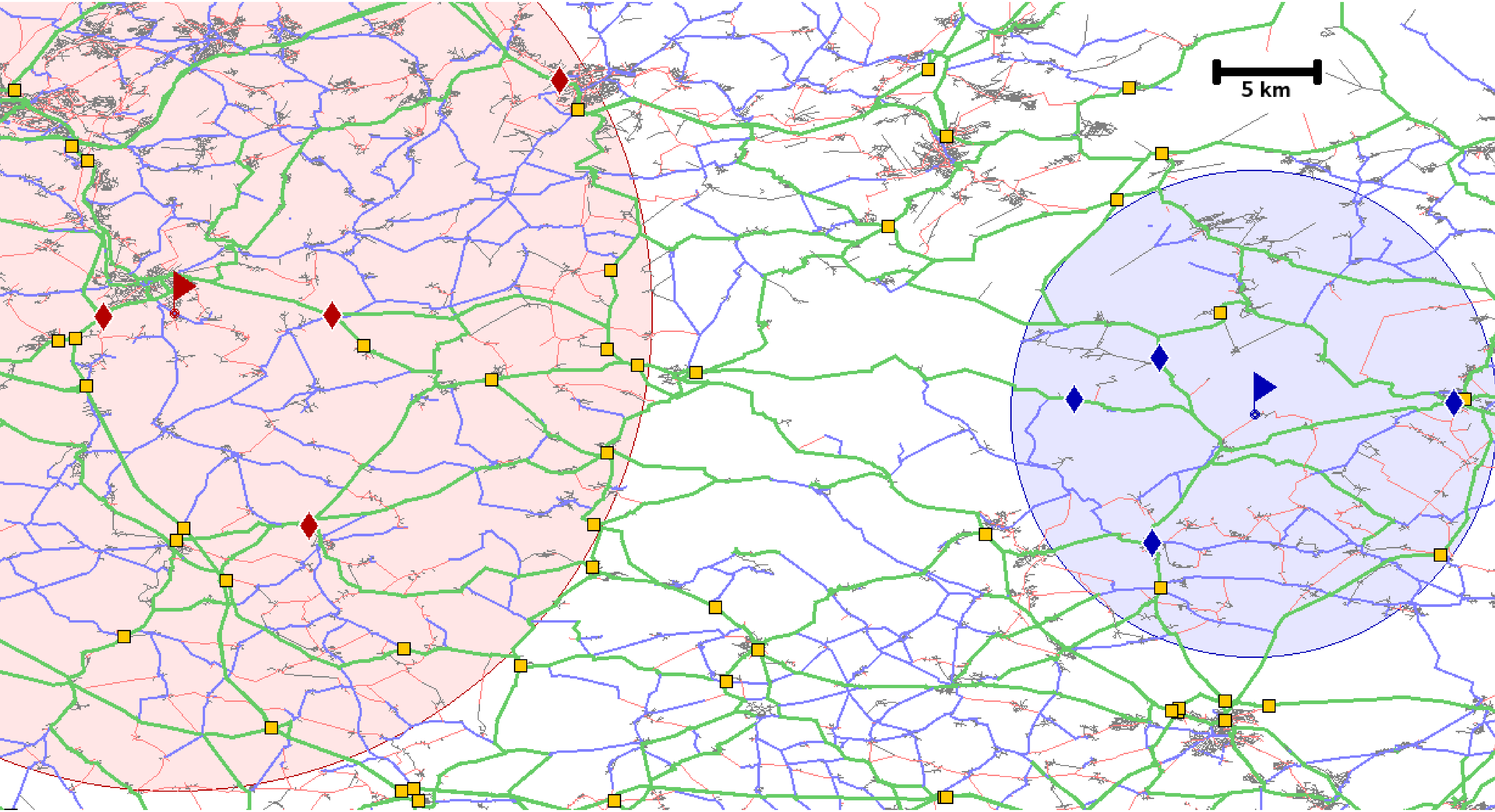
secondary distance table: store only distances between "nearby" secondary transit-nodes.

generalized many-to-many

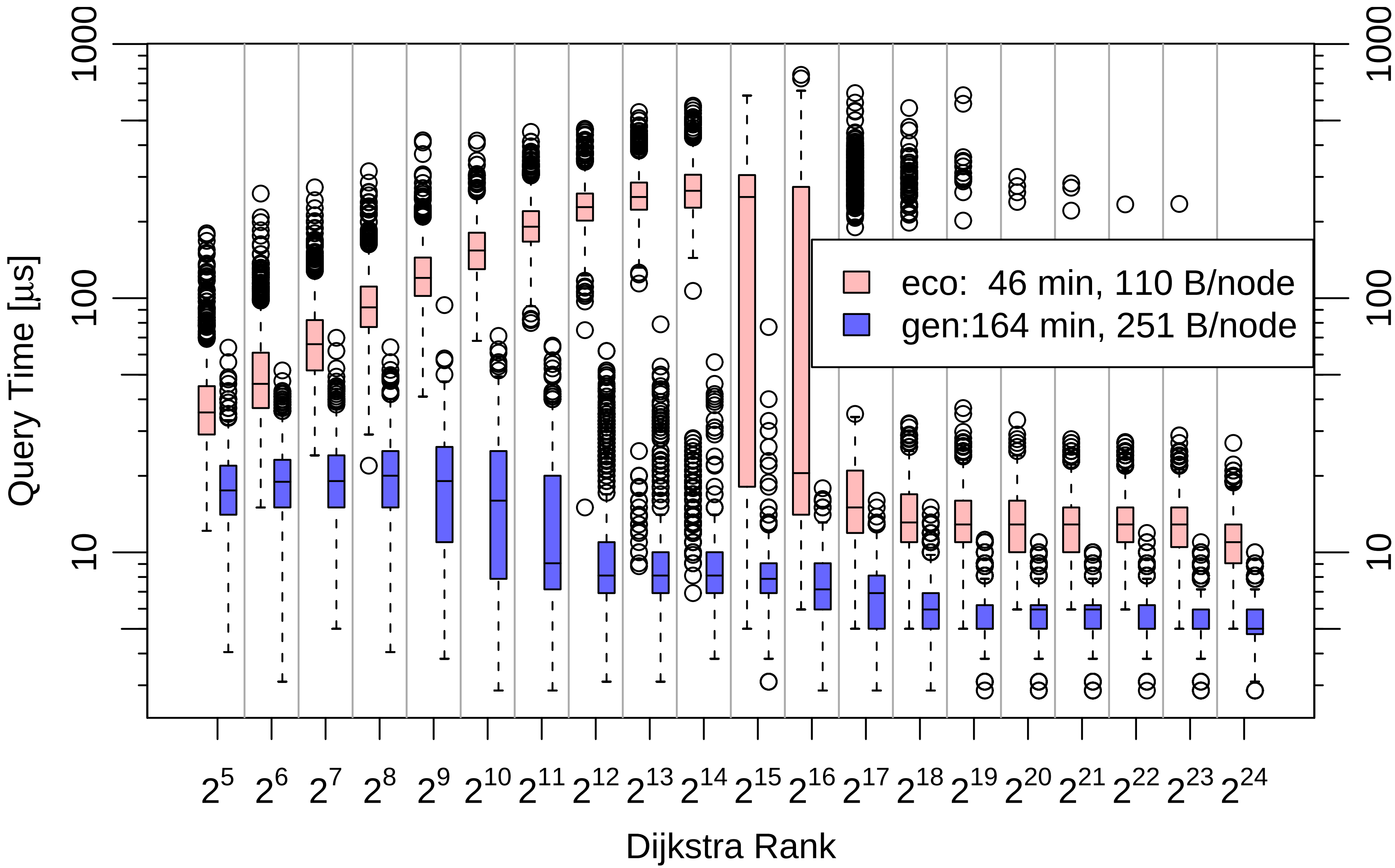
...secondary locality filter, tertiary transit-nodes,...

Base case: very limited local search

Example



Local Queries (Transit-Node Routing, Europe)



Summary

Highway Hierarchies: Fast routing, fast preprocessing, low space, few tuning parameters, basis for many-to-many, transit-node routing, highway-node routing. stay tuned

Many-to-Many: Huge distance tables are tractable.
Subroutine for transit-node routing.

Transit-Node Routing: Fastest routing so far.

Summary: A Horse-Race Perspective

method	first pub.	date mm/yy	size $n/10^6$	space Byt$/n$	preproc. [min]	speedup
separator multi-level	[SWW99]	04/99	0.1	?	$>$ 5 400	52
edge flags (basic)	[Lau04]	03/04	6	13	299	523
landmark A^*	[GolHar05]	07/04	(18)	72	13	28
edge flags	[KMS05]	01/05	1	141	2 163	1 470
HHs (basic)	[SS05]	04/05	18	29	161	2 645
adv. reach	[GKW06]	10/05	18	82	1 625	1 559
	[GKW06]	08/06	18	32	144	3 830
adv. HHs	[DSSW06]	08/06	18	76	22	11 496
high-perf. multi-level	[Mul06]	06/06	18	181	11 520	401 109
transit nodes (gen)	[BFMSS07]	10/06	18	251	164	1 129 143
highway nodes (mem)	[SS07]	01/07	18	2	24	4 079

Summary: An Application Perspective

HH= highway hierarchy

Static low-cost mobile route planning: low space HHs

Static server-based: transit-node routing

Logistics: Many-to-many HHs (HNR when edge weights change often)

Microscopic Traffic Simulation: transit-node routing ?

Macroscopic Traffic Simulation: Many-to-many HHs

Future Work I: More on Static Routing

- Better choices for transit-node sets
  (use centrality measures, separators, explicit optimization,...)
- Better integration with goal directed methods.
  (PCDs, A^*, edge flags, geometric containers)
- Experiments with other networks.
  (communication networks, VLSI, social networks, computer games, geometric problems, ...)

Future Work II: Theory Revisited

- Correctness proofs
- Stronger impossibility results (worst case)
- Analyze speedup techniques for model graphs
- Characterize graphs for which a particular (new?) speedup technique works well
- A method with low worst-case query time, but preprocessing might become quadratic ?

Future Work III: Towards Applications

- Turn penalties (implicitly represented)

  Just bigger but more sparse graphs ?

- Parallelization (server scenarios, logistics, traffic simulation)

  easy (construction, many-to-many, many queries)

- Mobile platforms

  $\rightsquigarrow$ adapt to memory hierarchy (RAM $\leftrightarrow$ flash)

  $\rightsquigarrow$ data compression

Future Work IV: Beyond Static Routing

- Dynamic routing (e.g. for transit-node routing) stay tuned
- Time-dependent networks
  (public transportation, traffic-dependent travel time)
- Preprocessing for an entire spectrum of objective functions
- Multi-criteria optimization
  (time, distance, fuel, toll, driver preferences,. . .)
- Approximate traffic flows
  (Nash-equilibria, (fair) social optima)
- Traffic steering (road pricing, . . .)
- Stochastic optimization

An Algorithm Engineering Perspective

Models: Preprocessing, point-to-point, dynamic, many-to-many

parallel, memory hierarchy, time dependent, multi-objective,. . .

Design: HHs, HNR, transit nodes,. . . wide open

Analysis: Correctness, per instance. big gap

Implementation: tuned, modular, thorough checking, visualization.

Experiments: Dijkstra ranks, worst case, cross method. . . .

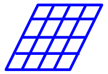
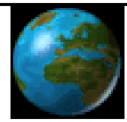
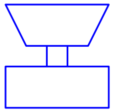
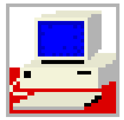
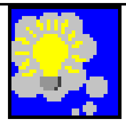
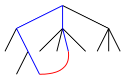
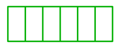
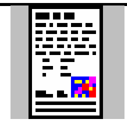
Instances: Large real world road networks.

turn penalties, queries, updates, other network types

Algorithm Libraries: ???

Applications: Promising contacts, hiring. more should come.

Gaps Between Theory & Practice

Theory		$\longleftrightarrow$		**Practice**
simple		**appl. model**		complex
simple		**machine model**		complex
complex		**algorithms**	FOR	simple
complex		**data structures**		simple
worst case	max	**complexity measure**		inputs
asympt.	$\mathcal{O}(\cdot)$	**efficiency**	42%	constant factors

Goals

- bridge gaps between theory and practice
- accelerate transfer of algorithmic results into applications
- keep the advantages of theoretical treatment:

  generality of solutions and

  reliabiltiy, predictabilty from performance guarantees

Canonical Shortest Paths

$\mathcal{SP}$: Set of shortest paths

$\mathcal{SP}$ canonical $\Leftrightarrow$

$$\forall P = \langle s, \ldots, s', \ldots, t', \ldots, t \rangle \in \mathcal{SP} : \langle s' \rightarrow t' \rangle \in \mathcal{SP}$$

A Meaning of "Local"

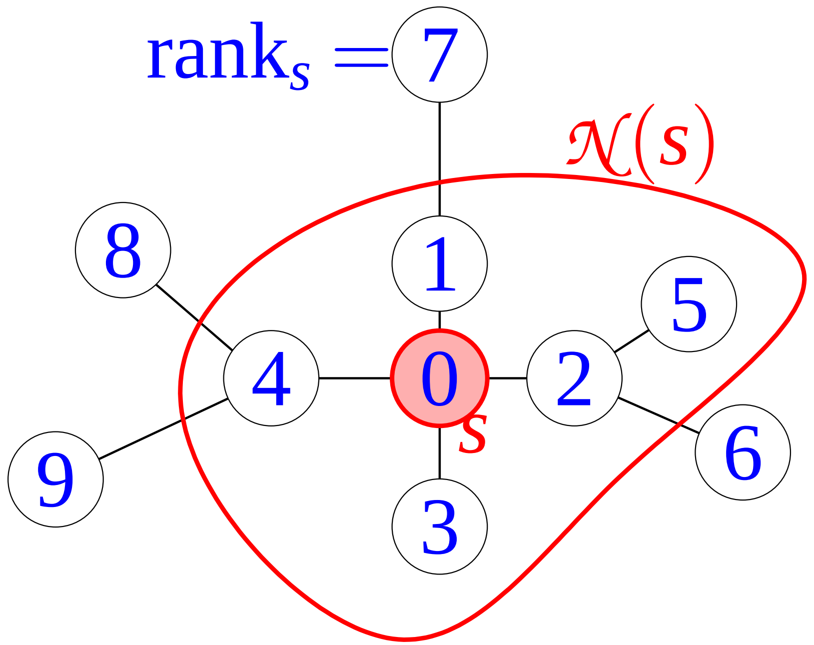
- choose neighbourhood radius $r(s)$

  e.g. distance to the H-closest node for a fixed parameter H

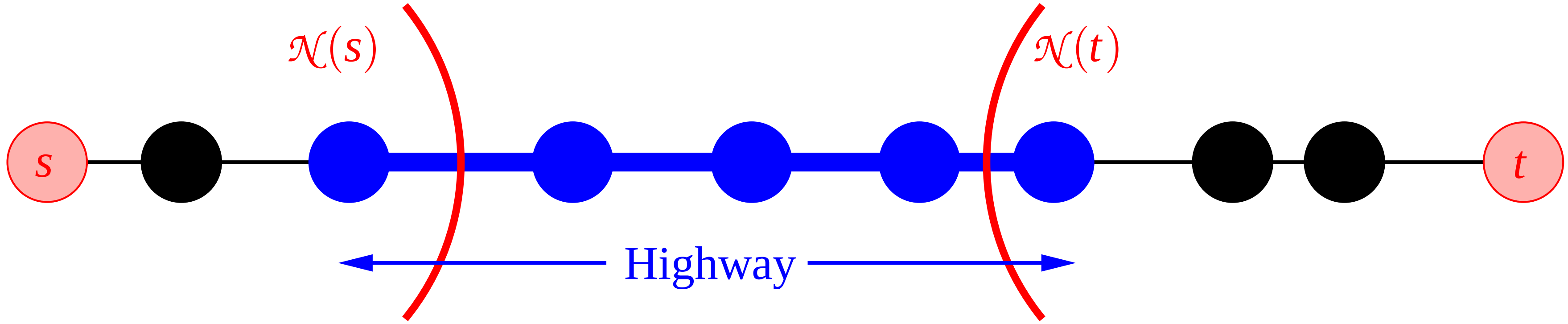
- define neighbourhood of s:

  $\mathcal{N}(s) := \{v \in V \mid d(s,v) \leq r(s)\}$

- example for $H = 5$



Highway Network



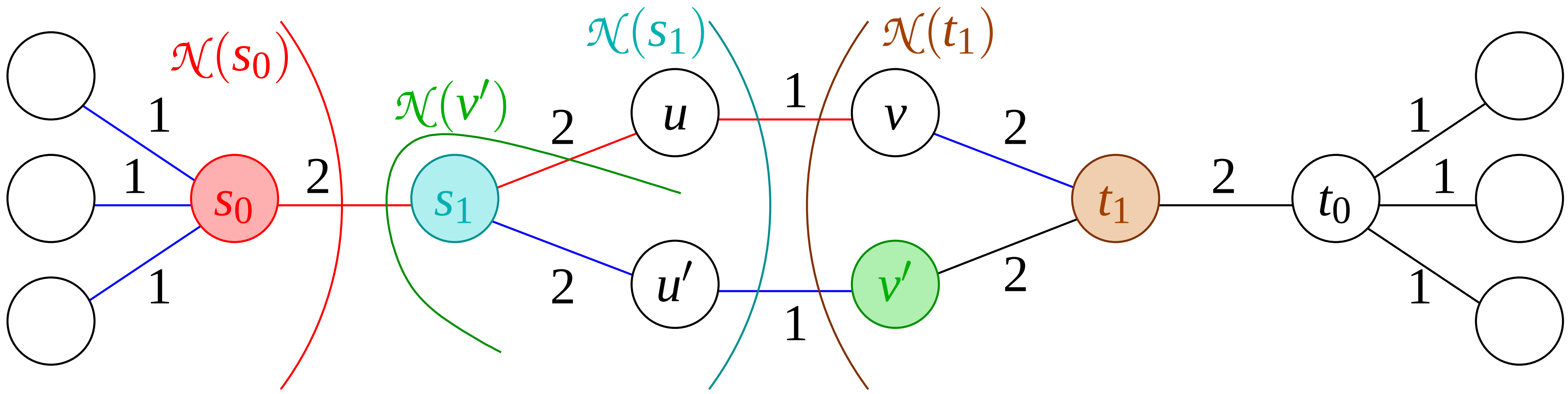
Edge (u,v) belongs to highway network *iff* there are nodes s and t s.t.

- (u,v) is on the "*canonical*" shortest path from s to t

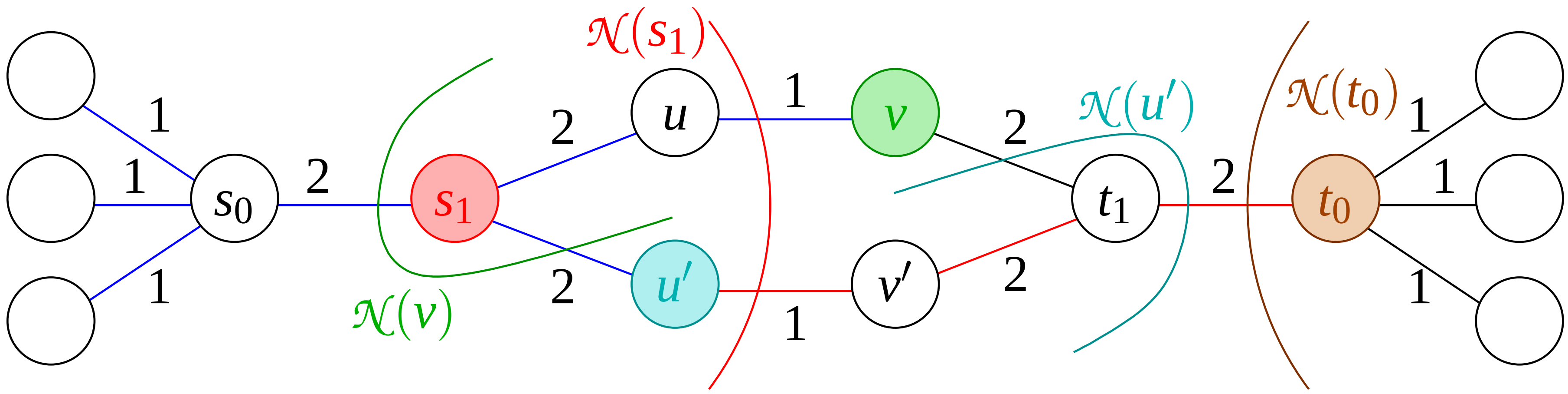
and

- (u,v) is not entirely within $\mathcal{N}(s)$ or $\mathcal{N}(t)$

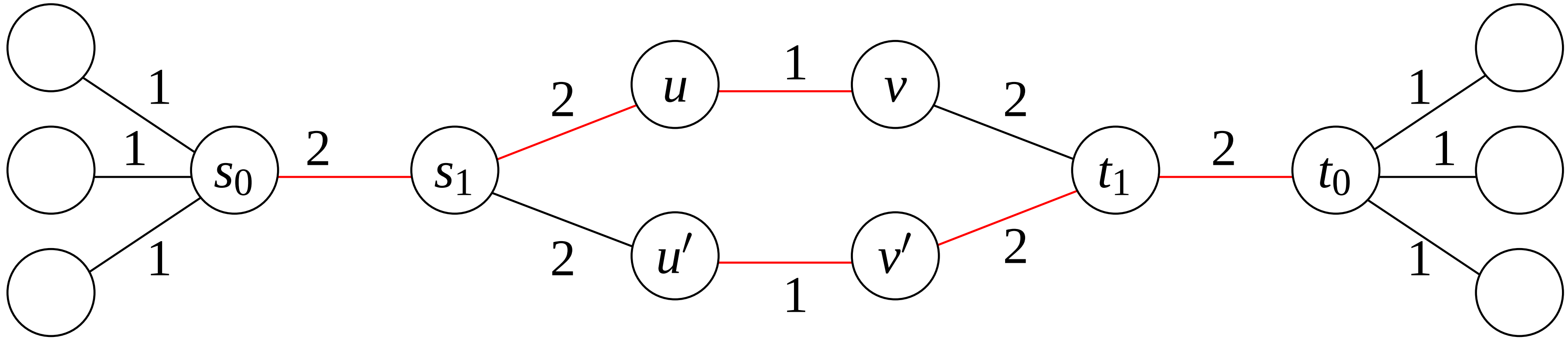
Canonical Shortest Paths



(a) Construction, started from s_0.

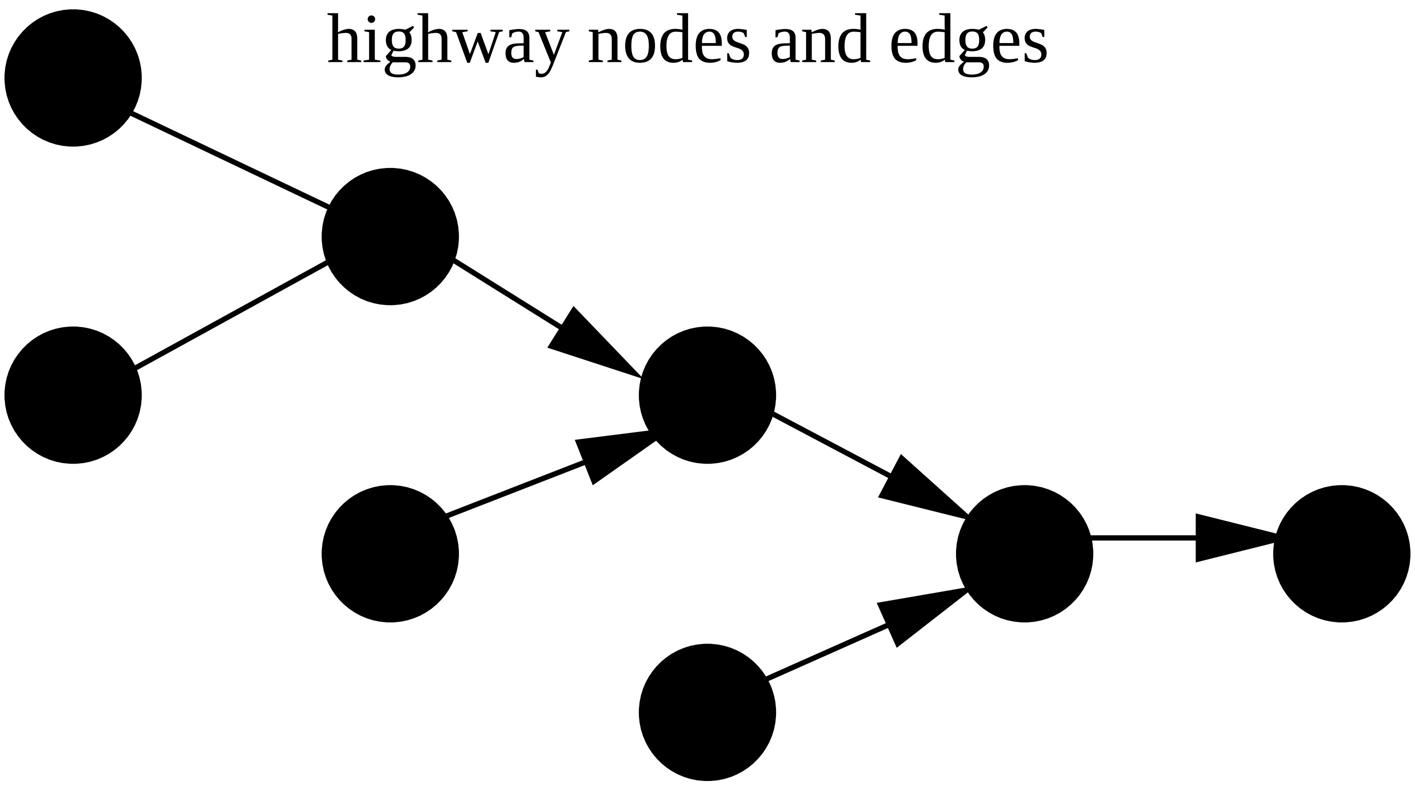


(b) Construction, started from s_1.

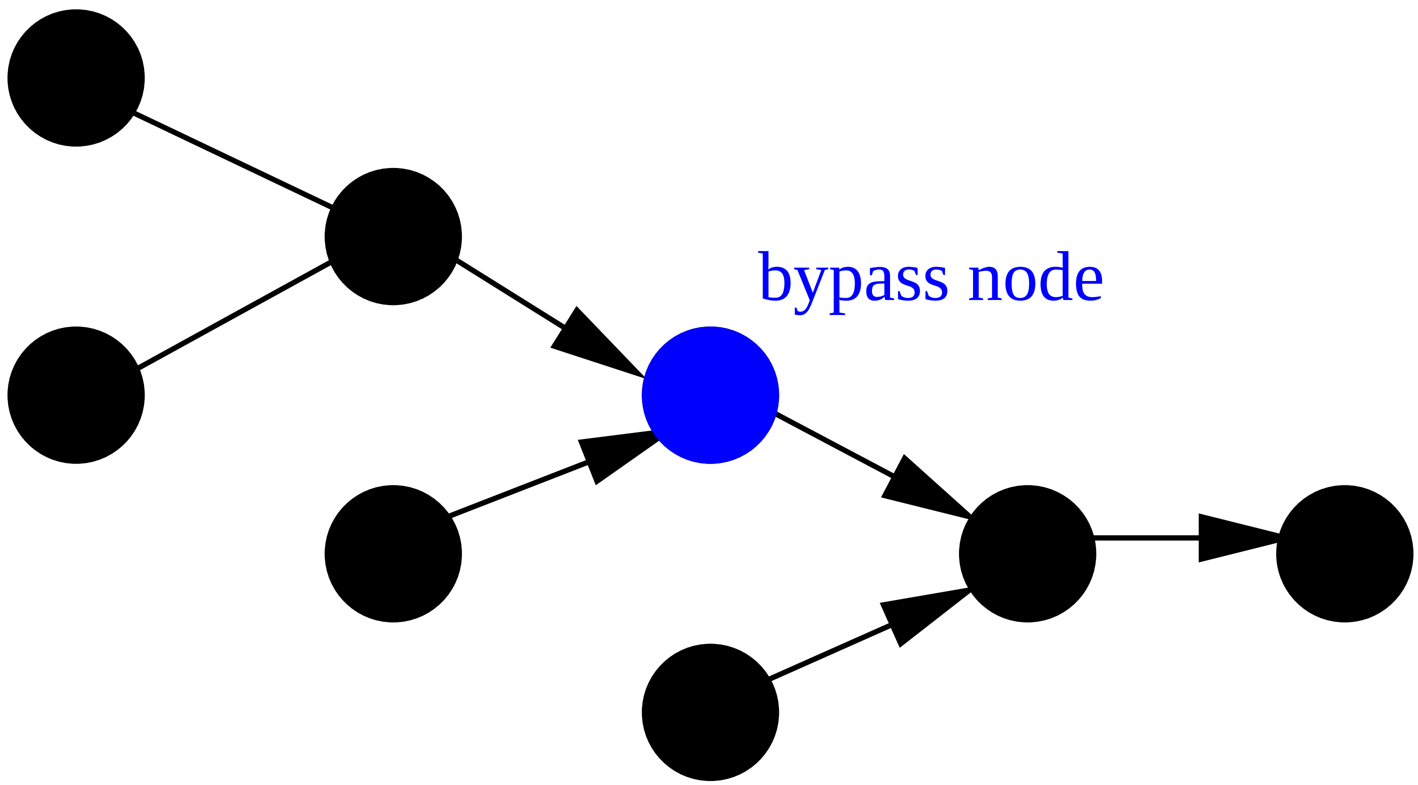


(c) Result of the construction.

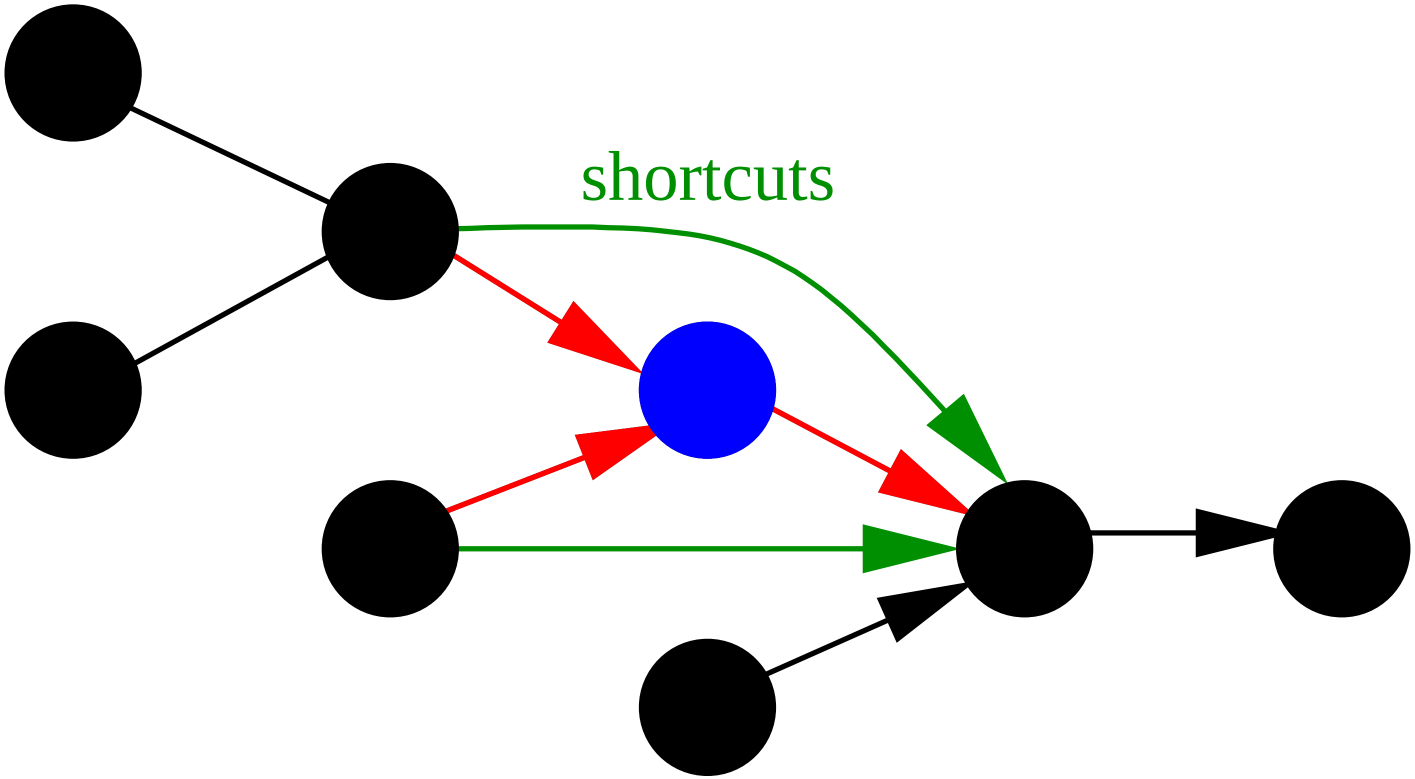
Contraction



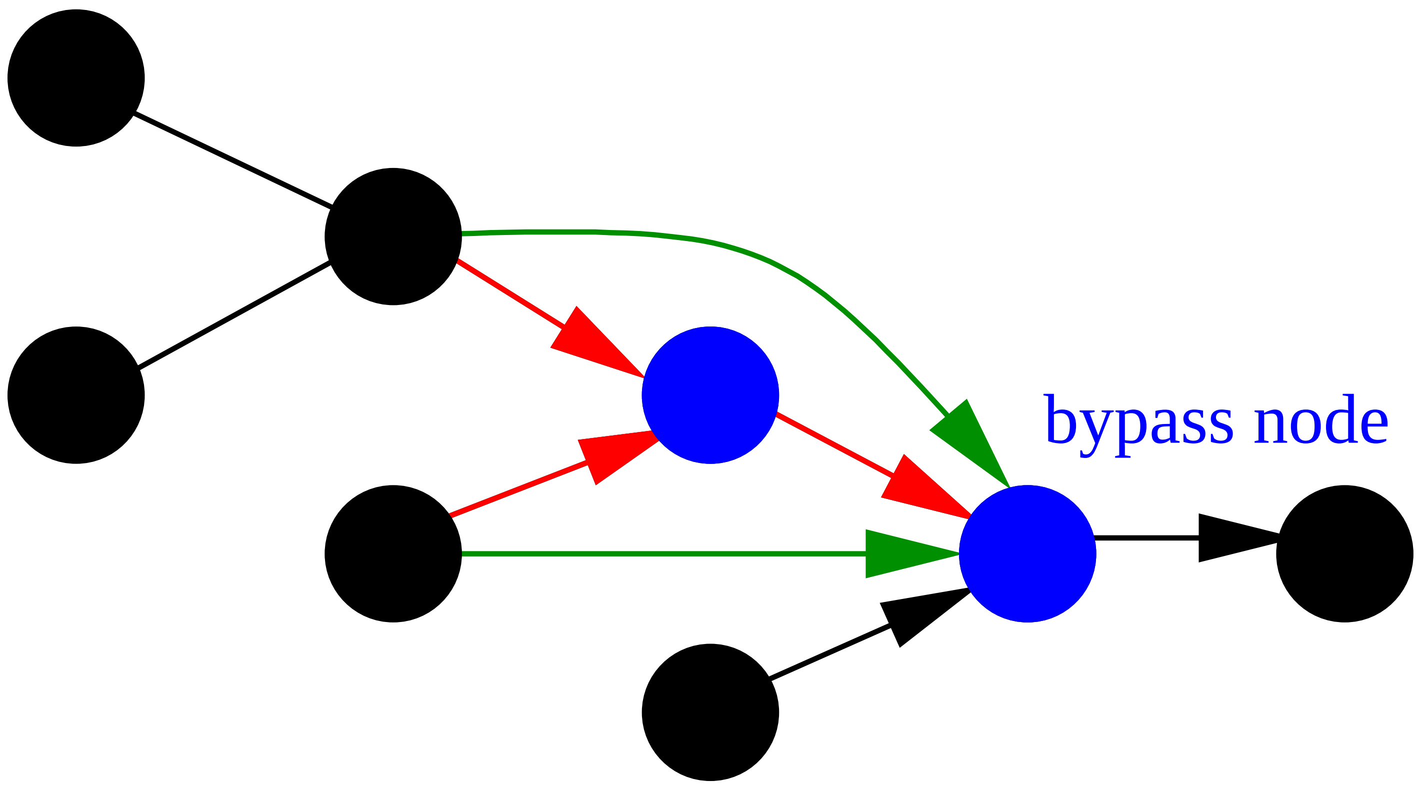
Contraction



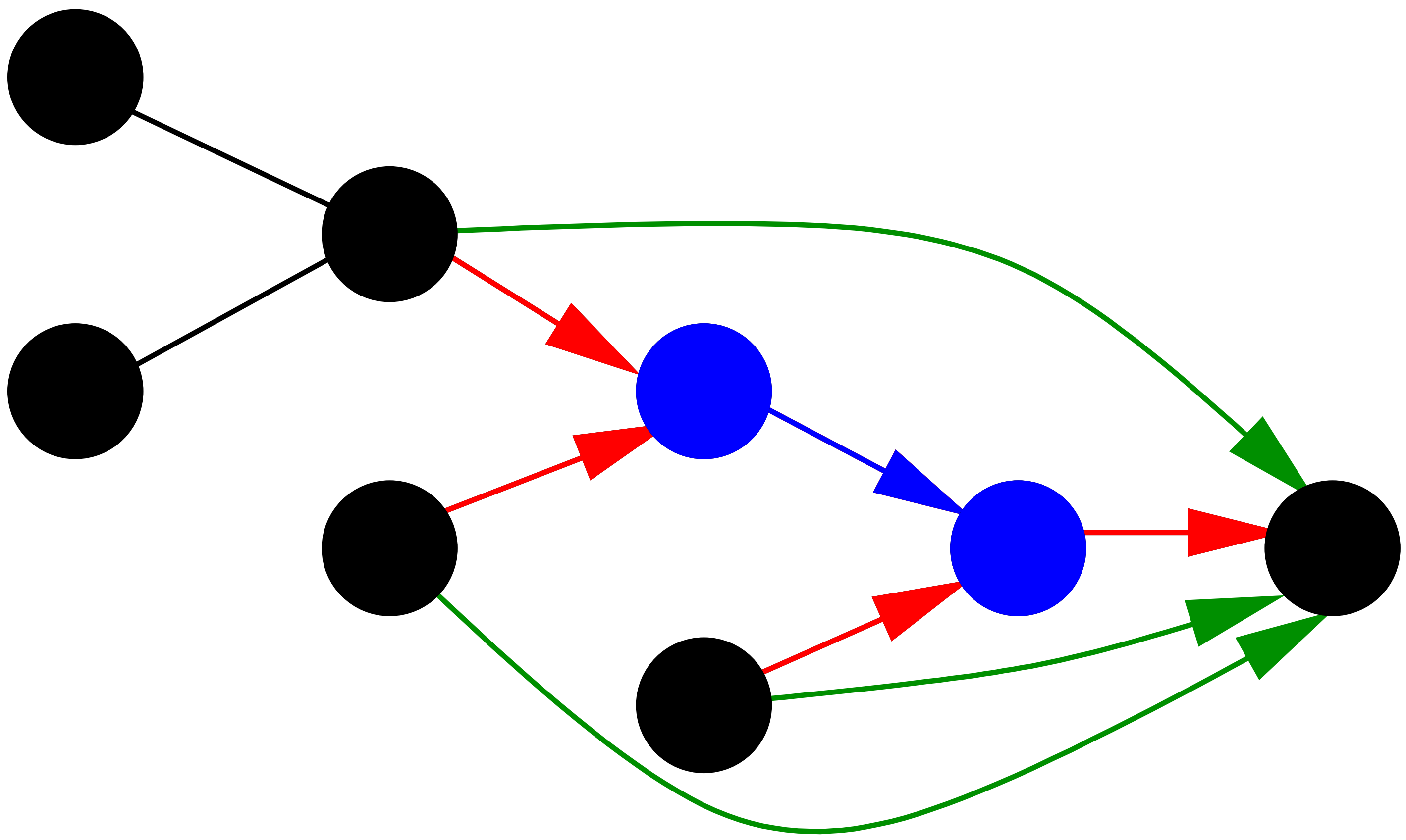
Contraction



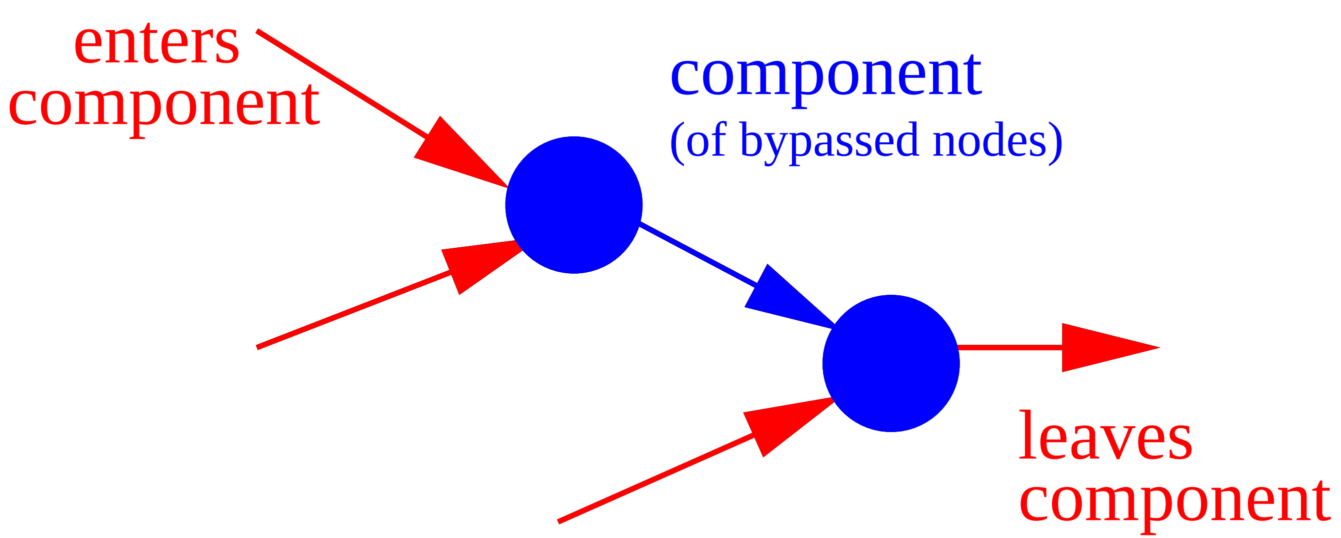
Contraction



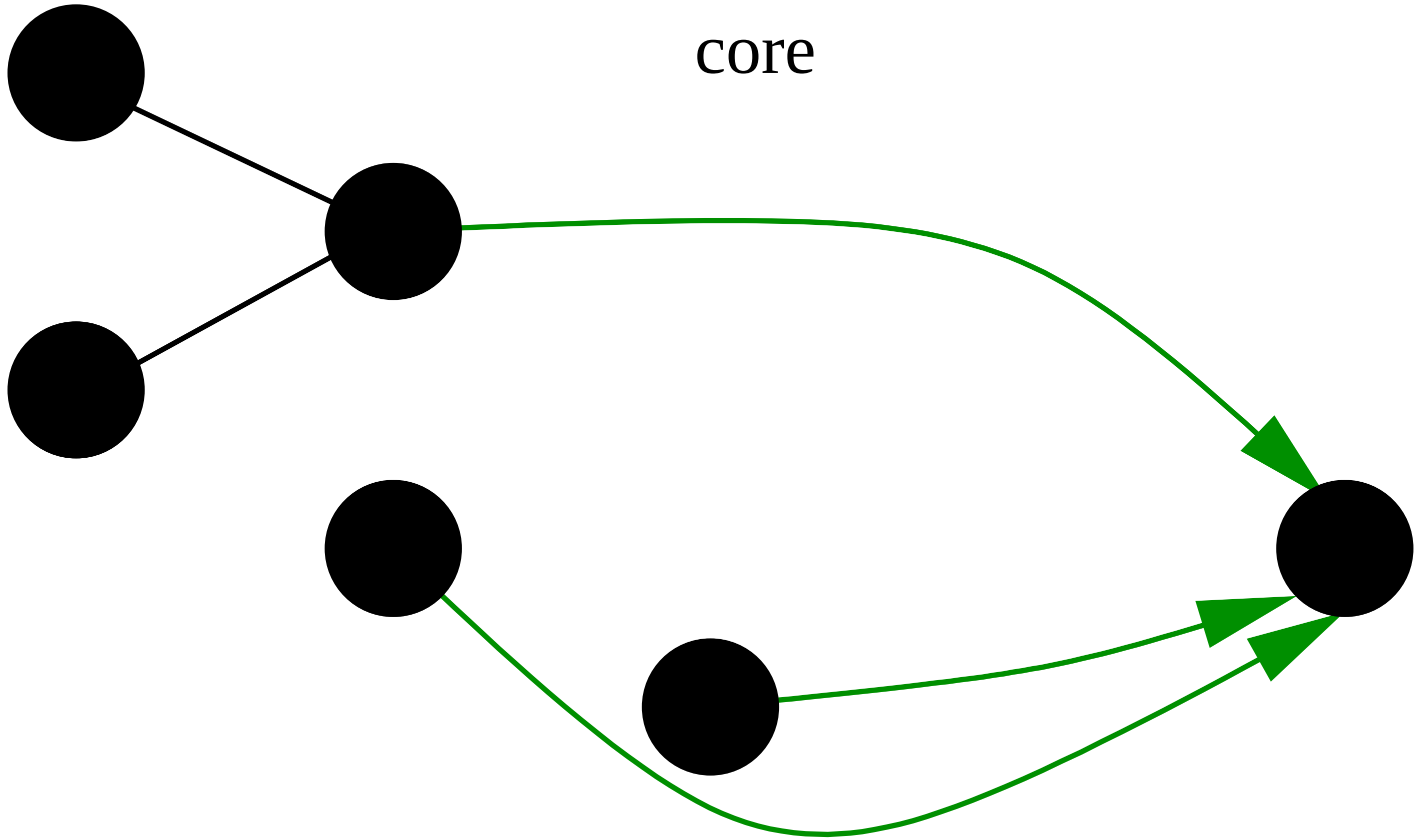
Contraction



Contraction



Contraction



Contraction

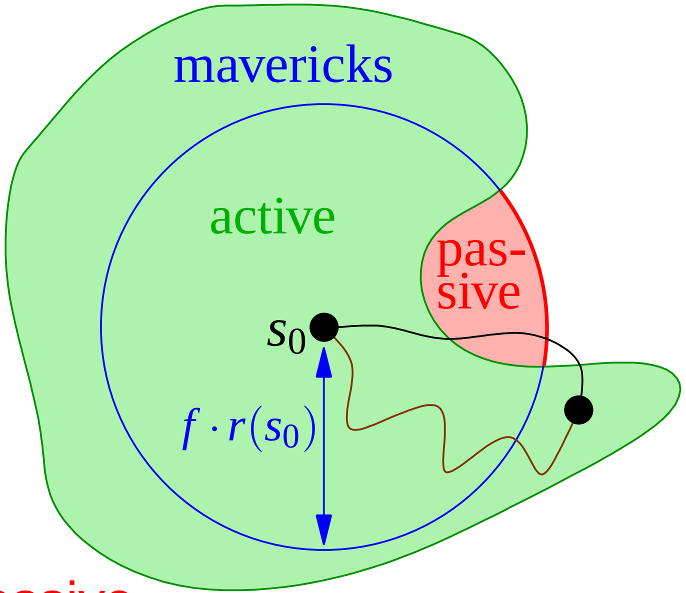
Which nodes should be bypassed?

Use some heuristic taking into account

- the number of shortcuts that would be created and
- the degree of the node.

Fast Construction of the Highway Network

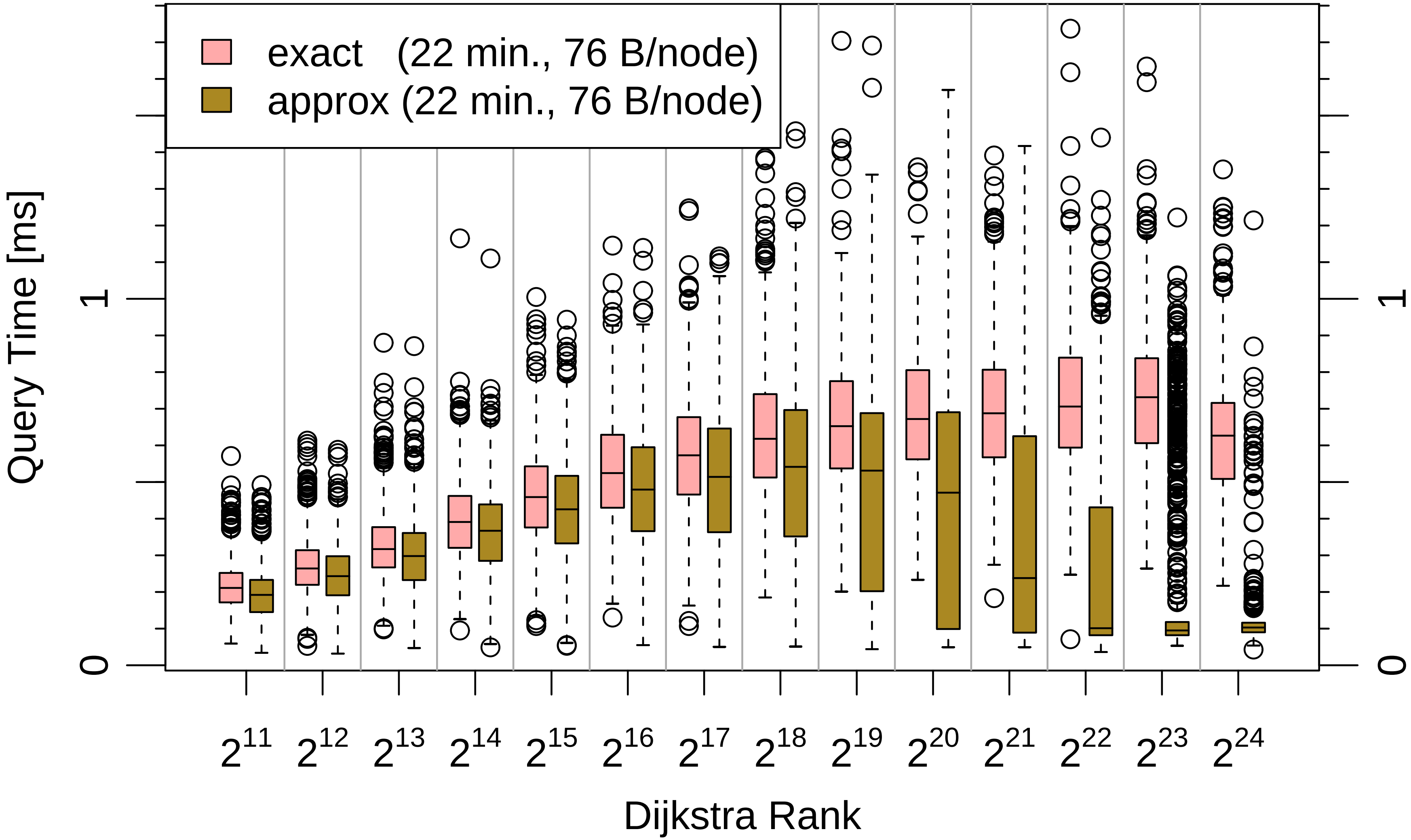
Look for HH-edges only in (modified) local SSSP search trees.



- Nodes have state active, passive, or mavericks.
- s_0 is active.
- Node states are inherited from parents in the SSSP tree.
- abort condition$(p) \longrightarrow p$ becomes passive.
- $d(s_0, p) > f \cdot r(s_0) \longrightarrow p$ becomes maverick.
- all nodes maverick? $\longrightarrow$ stop searching from passive nodes
- all nodes passive or maverick? $\longrightarrow$ stop

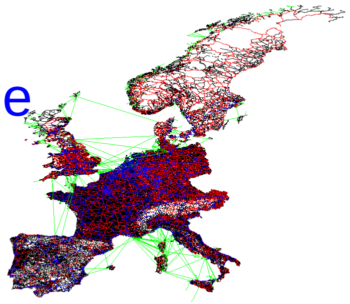
Result: superset of highway network

Local Queries (Highway Hierarchies Star, Europe)



Simple Solutions

Example: $10\,000 \times 10\,000$ table in Western Europe



- apply SSSP algorithm (e.g. DIJKSTRA) $|S|$ times $\approx 10\,000 \times 10\,\mathrm{s} \approx$ one day
- apply P2P algorithm (e.g. highway hierarchies[1]) $|S| \times |T|$ times $\approx 10\,000^2 \times 1\,\mathrm{ms} \approx$ one day

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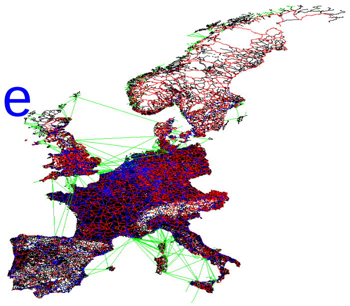
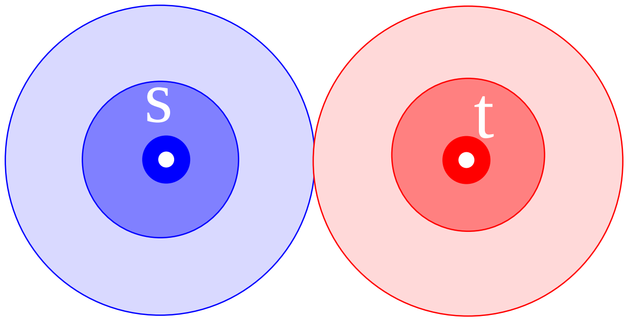
[1] requires about 15 minutes preprocessing time

Our Solution

Example: 10 000 $\times$ 10 000 table in Western Europe



- many-to-many algorithm
  based on highway hierarchies[1]



$\approx$ one minute

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[1] requires about 15 minutes preprocessing time

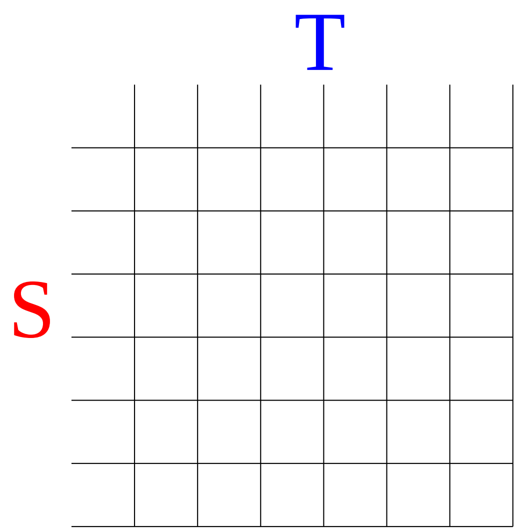
Main Idea

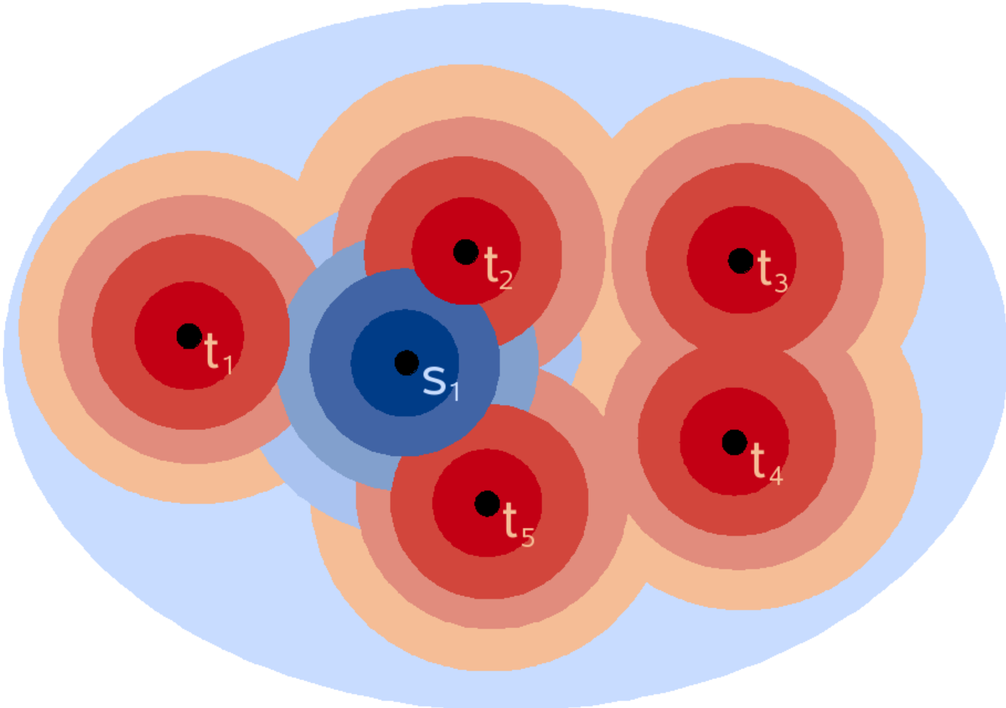
- instead of $|S| \times |T|$ bidirectional highway queries
- perform $|S| + |T|$ unidirectional highway queries

Algorithm

- maintain an $|S| \times |T|$ table D of tentative distances

  (initialize all entries to ∞)



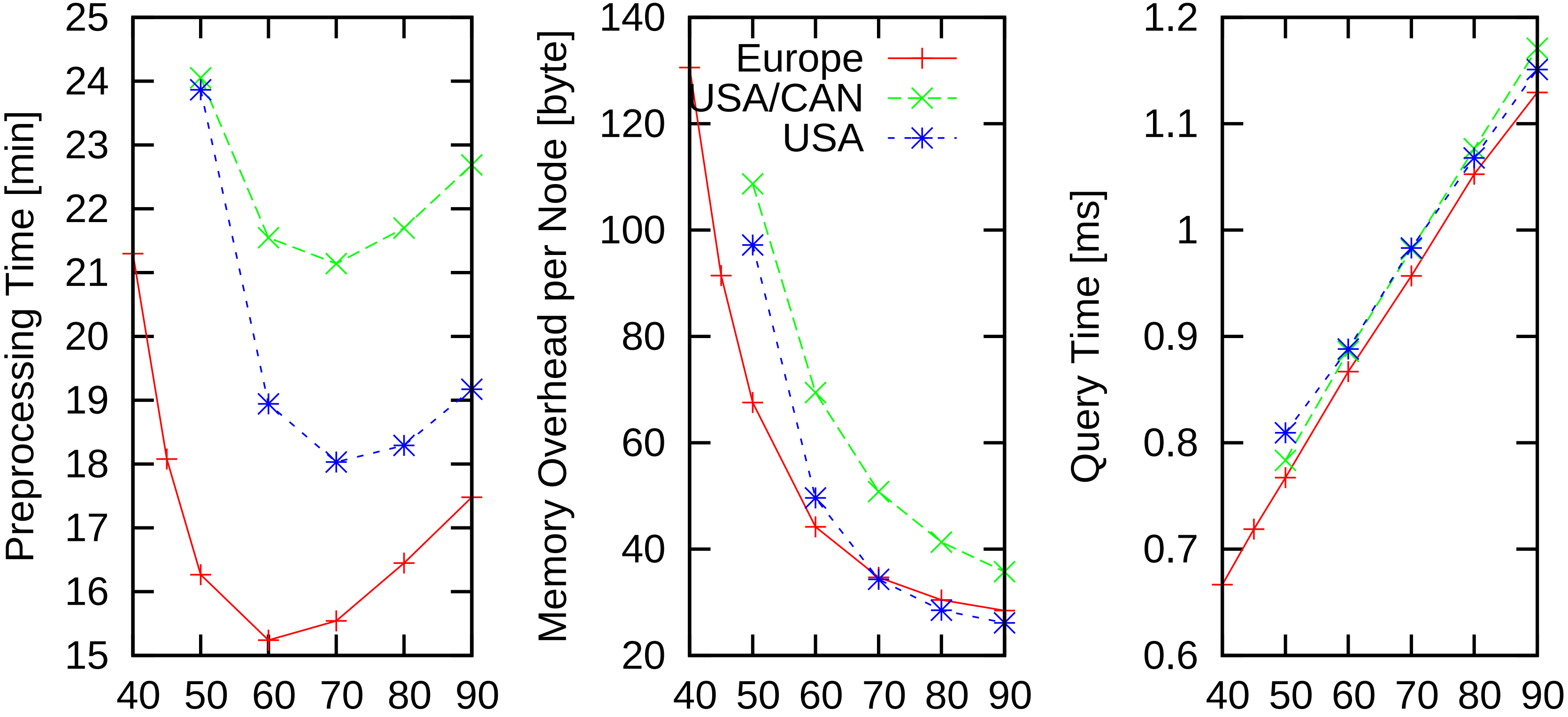


- for each $t \in T$, perform backward search
  store search space entries $(t,u,d(u,t))$
- arrange search spaces: create a bucket for each u
- for each $s \in S$, perform forward search
  at each node u, scan all entries $(t,u,d(u,t))$ and
  compute $d(s,u)+d(u,t)$, update $D[s,t]$

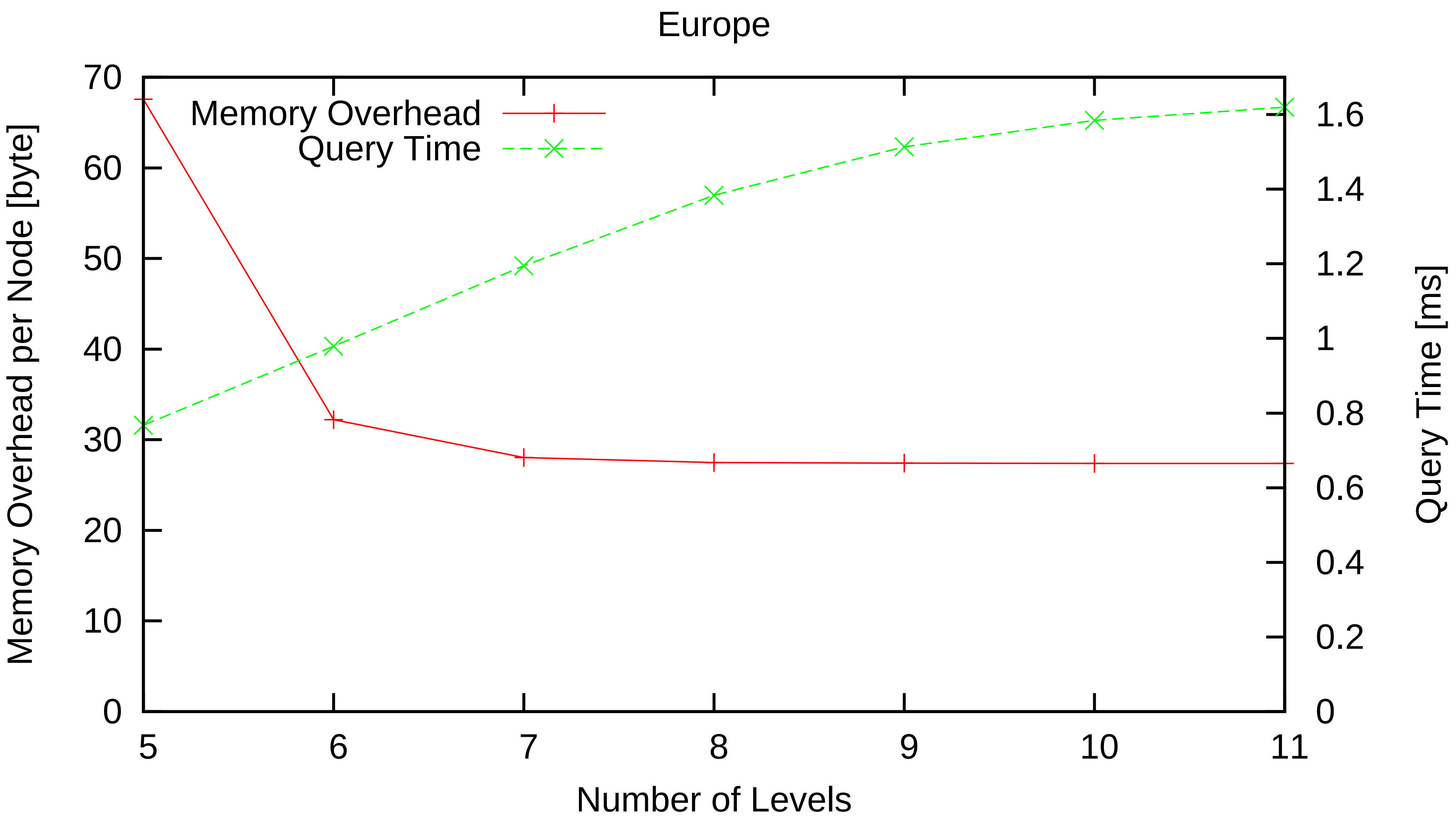
Different Combinations

metric		$\emptyset$	DistTab	ALT	both	
		Europe				
time	preproc. time [min]	17	19	20	22	
	total disk space [MB]	886	1 273	1 326	1 714	
	#settled nodes	1 662	916	916	686	(176)
	query time [ms]	1.16	0.65	0.80	0.55	(0.18)
dist	preproc. time [min]	47	47	50	49	
	total disk space [MB]	894	1 506	1 337	1 948	
	#settled nodes	10 284	5 067	3 347	2 138	(177)
	query time [ms]	8.21	4.89	3.16	1.95	(0.25)

Neighbourhood Size



Number of Levels



Contraction Rate