Rainbow Sort

– Sorting at the Speed of Light –

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Abstract

Rainbow Sort is an unconventional method for sorting, which bases on the physical concepts of *refraction* and *dispersion*. It is inspired by the observation that light that traverses a prism is *sorted* by wavelength. This property is used to design a sorting method that has the potential of running in $\Theta(n)$ with a space complexity of $\Theta(n)$, where n denotes the number of elements that are sorted.

In Section 1, some upper and lower bounds for sorting are presented in order to provide a basis for comparisons. In Section 2, the physical background is outlined, the setup and the algorithm are presented and a lower bound for Rainbow Sort of $\Omega(n)$ is derived. In Section 3, we describe essential difficulties that arise when Rainbow Sort is implemented. Particularly, restrictions that apply due to the Heisenberg uncertainty principle have to be considered. Furthermore, we sketch a possible implementation that leads to a running time of $O(n+m)$, where m is the maximum key value, i.e., we assume that there are integer keys between 0 and m. Section 4 concludes with a summary of the complexity and some remarks on open questions, particularly on the treatment of duplicates and the preservation of references from the keys to records that contain the actual data. In Appendix A, a simulator is introduced that can be used to visualize Rainbow Sort.

1 Introduction

Traditionally, there is the distinction between *comparison-based* and *non-comparison-based* sorting algorithms. Comparison-based means that the order of the elements is determined by several comparisons between pairs of elements. It is well known that the lower bound for comparison-based sorting algorithms is $\Omega(n \log n)$, where n denotes the number of elements that are sorted [CLRS01, p. 167]. There are several comparison-based algorithm that are optimal with respect to the asymptotic running time, for example Mergesort and Heapsort [CLRS01, p. 127]. While the running time of these algorithms is in $\Theta(n \log n)$, their space complexity is in $\Theta(n)$.

Non-comparison-based sorting algorithms do not compare the elements by pairs, but they deal with every single element and move it according to its key. Therefore, these algorithms assume that there are only integer keys in a distinct range from 0 to m. Counting Sort [CLRS01, p. 168] is an example for such an algorithm. Its runtime and its space complexity are in $O(n+m)$. The lower bound of $\Omega(n \log n)$ does not apply to non-comparison-based sorting algorithms. In this case, we have only the trivial lower bound of $\Omega(n)$ because, obviously, we have to look at each element at least once if we want to sort it.

In addition to these two traditional types, we can consider *unconventional* sorting methods, which are in many respects so different from the conventional methods that it is appropriate to list them separately. Bead Sort [ACD04] ranks among this third type of sorting methods. In the Bead Sort algorithm each integer x is represented by x beads that are arranged in a horizontal row on vertical rods. When the input is given in this way, the beads fall down until they hit the bottom or another bead that already rests. If each row with x beads is again interpreted as the number x, the rows represent the sorted data after all beads have reached their final position. The processing time of Bead Sort is in $O(\sqrt{n})$ because the falling beads are accelerated due to gravity so that the duration of the fall is $\sqrt{2h/g}$, where the height h is proportional to n and g is

a constant, namely the acceleration of gravity. However, in this case we have to regard the input and output complexity of $\Theta(n)$ as well because it dominates over the processing time. Hence, the total running time is in $\Theta(n)$. Unfortunately, the space complexity of Bead Sort is in $\Theta(n \cdot m)$, which is a disadvantage. Table 1 summarizes the complexities of several sorting methods.

Type	Method	Input	Processing	Output	Σ
comparison-based	Heapsort	n	$n \log n$	n	$\Theta(n \log n)$
non-comparison-based	Counting Sort	n	$n + m$	n	$\Theta(n + m)$
unconventional	Bead Sort	n	$\sqrt{n}$	n	$\Theta(n)$

Table 1: Complexities of several sorting methods

Let us have a look at the gap between the lower and the upper bounds of sorting. For comparison-based sorting, there is no such gap because the upper bounds of known algorithms like Heapsort match with the lower bound of $\Omega(n \log n)$. For non-comparison-based sorting, there is no known conventional algorithm that reaches the trivial lower bound of $\Omega(n)$. However, Bead Sort accomplishes this goal, even though the space complexity of $\Theta(n \cdot m)$ is inconvenient. Hence, there is still the question if it is possible to sort in $\Theta(n)$ using a space complexity of less than $\Theta(n \cdot m)$.

2 Idea

2.1 Basic Physical Concepts

Rainbow Sort bases on the basic physical concepts *refraction* and *dispersion*. Here, we give only a brief outline of these concepts as far as it is required in order to be able to describe the idea of Rainbow Sort. For more detailed descriptions the reader may refer to [Hec02].

When a light ray that moves through vacuum enters a transparent material with a refraction index[1] $n > 1$, it gets refracted as illustrated in Figure 1. The extent of the refraction can be expressed by *Snell's Law* [Hec02, p. 101]:

$$\frac{\sin \alpha}{\sin \beta} = \frac{c}{c_n} = n, \tag{1}$$

where the refraction index n corresponds with the ratio of the speed of light c in vacuum to the speed of light c_n in the material with the refraction index n [Hec02, p. 66].

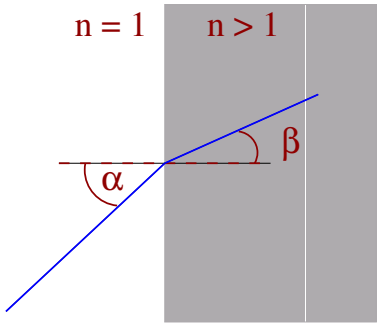

Figure 1: Refraction

When a ray traverses a prism, it is refracted twice, once when it enters the prism and once when it leaves it. The *angular deviation* δ denotes the angle between the incoming and the outgoing ray.

[1] As n is the common identifier of the number of elements *and* of the refraction index, the author decided to use n for both values. The meaning of n will always be clear from the context.

For the symmetrical optical path, i.e., a special case where the ray traverses the prism parallel to the base line, a simple formula for the angular deviation can be derived [Hec02, p. 188]:

$$\delta = 2 \arcsin\left(n \sin\frac{\varepsilon}{2}\right) - \varepsilon,$$

where ε is the angle of the prism opposite to the base line. Figure 2 represents this special case.

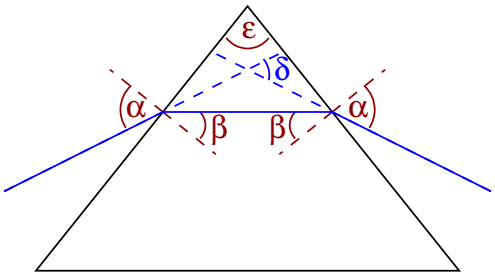

Figure 2: Symmetrical optical path through a prism

Due to the fact that the arcsin-function is strictly monotonic increasing, we can conclude that

$$\text{the greater the refraction index } n\text{, the greater the angular deviation } \delta. \tag{2}$$

For the general case, the formula gets more complicated, but this fact still holds.

The fact that the refraction index n depends on the wavelength λ of the ray is called *dispersion*. Figure 3 shows the refraction index of crown glass depending on the wavelength.

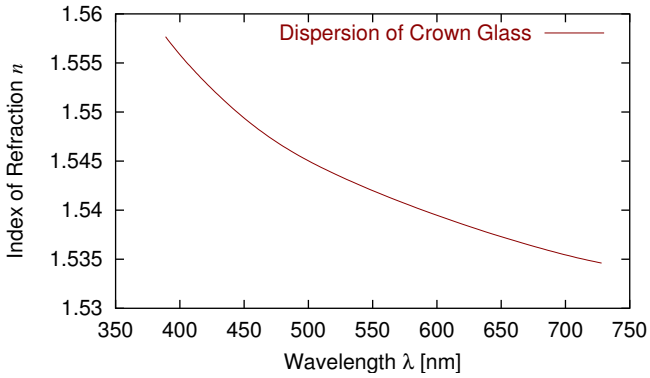

Figure 3: Dispersion of crown glass[2]

We can notice that

$$\text{the less the wavelength } \lambda\text{, the greater the refraction index } n. \tag{3}$$

2.2 Setup and Algorithm

Figure 4 represents the basic setup of Rainbow Sort. It can be divided into an input, a processing, and an output part.

- The input part consists of a light source. The unsorted input data is encoded into wavelengths: each number of the input is mapped to a wavelength. The light source generates a ray whose spectrum consists exactly of the wavelengths that represent the input data.

[2]based on measurement data taken from [Hec02, p. 70]

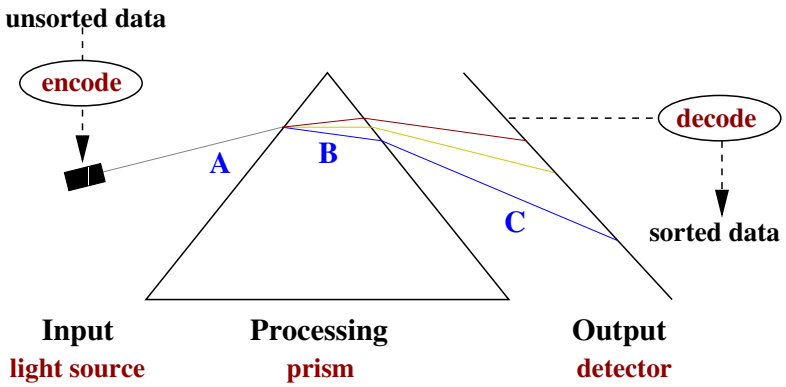

Figure 4: Setup of Rainbow Sort

- The processing part consists of a prism which the generated ray is sent through. Due to the refraction and dispersion, the ray is split into n monochromatic rays that are sorted by wavelength.
- On the output side a detector receives the incoming rays. It is positioned in such a way that the length of the path from the prism is maximal for the minimum wavelength and minimal for the maximum wavelength. The detector decodes the incoming rays and outputs the sorted data.

In order to encode the input data, a strictly monotonic increasing function $f : \{0, 1, 2, \ldots, m\} \to [\lambda_{min}, \lambda_{max}]$ is used. It maps from the set of keys to a range of wavelengths that is fixed for any specific implementation. Since the spectrum of light is continuous, we can map to any real number in $[\lambda_{min}, \lambda_{max}]$. Actually, a mapping to rational numbers is sufficient in order to construct such a function f for any given m. For decoding we use the inverse function f^{-1}, which is strictly monotonic increasing, too. So far, this approach cannot deal with duplicates, i.e., with the case that the same number x appears more than once in the input data. Section 4 discusses some possibilities of the treatment of duplicates. However, for now we assume that there are no duplicates for the sake of simplicity.

Figure 5 contains the Rainbow Sort algorithm.

Input
`Ray` $ray := \emptyset$
for each $x \in$ *Input* **do** $ray := ray \cup f(x)$

Processing
send ray through prism

Output
`Stack` $sorted := \emptyset$
`Wavelength` $cur\lambda := \infty$
whenever $\min \lambda(incoming\ rays) < cur\lambda$ **do**
    $cur\lambda := \min \lambda(incoming\ rays)$
    $sorted$.push$(f^{-1}(cur\lambda))$
    if $sorted$.size $= n$ **then return** $sorted$

Figure 5: Rainbow Sort algorithm

The ray is interpreted as a set of wavelengths (which is initially empty). For each number x of the unsorted input data, x is encoded and the resulting wavelength is added to the ray. In the processing step the ray is sent through the prism. The detection of the sorted wavelengths takes

place in a special type of loop: whenever the minimum wavelength of all incoming rays is less than the current wavelength (which is initially set to infinity), the current wavelength is updated and decoded; the resulting number is added to a stack (which is initially empty) that saves the sorted data. Finally, there is a check if all n rays have already arrived. If so, the algorithm terminates and the stack contains the result.

2.3 Correctness and Complexity

In order to prove the correctness of Rainbow Sort, we first show that the wavelengths arrive at the detector in decreasing order: let $\lambda_{min} \leq \lambda_1 < \lambda_2 \leq \lambda_{max}$. Then λ_2 arrives before λ_1.

Proof:

$$\begin{aligned} & \lambda_1 < \lambda_2 \\ \overset{(3)}{\rightsquigarrow}\ & n(\lambda_1) > n(\lambda_2) \\ \overset{(2),(1)}{\rightsquigarrow}\ & \delta(\lambda_1) > \delta(\lambda_2) \quad (\rightsquigarrow \text{ longer path for } \lambda_1) \\ \wedge\ & c(\lambda_1) < c(\lambda_2) \quad (\rightsquigarrow \lambda_1 \text{ slower in the prism}), \end{aligned}$$

where $n(\lambda)$, $\delta(\lambda)$, and $c(\lambda)$ denote the refraction index, the angular deviation, resp. the speed of light in the prism depending on the wavelength λ. Table 2 compares the length of the path and the speed of λ_1 and λ_2.

path / speed	λ_1	λ_2
A	equal / equal	equal / equal
B	longer / slower	shorter / faster
C	longer / equal	shorter / equal

Table 2: Comparison of the length of the path and the speed of two wavelengths $\lambda_1 < \lambda_2$ in the sections A, B, and C as labeled in Figure 4

Before the ray enters the prism (Section A, cp. Figure 4), there is no difference between λ_1 and λ_2. In the prism (B), λ_1 is slower and has to cover a longer distance than λ_2. After the rays have left the prism (C), they move at the same speed, but again λ_1 has to cover a longer distance until it hits the detector. Hence, the whole setup is stacked against λ_1 so that λ_2 arrives first. □

Now, we can complete the **proof of correctness:** The fact that the wavelengths arrive in decreasing order leads to the following observation: whenever a new wavelength arrives, it is smaller than $cur\lambda$. Thus, for each new incoming ray, there is an iteration of the whenever-loop; inside the loop, the new incoming wavelength is assigned to $cur\lambda$. Hence, $cur\lambda$ equals to *all* wavelengths one after the other *in decreasing order*. Therefore, the output is *complete*, i.e., it consists of the same elements as the input, because the encoding function f is injective. Furthermore, we can conclude that the output is *sorted* in decreasing order because the decoding function f^{-1} is strictly monotonic increasing and hence, the existing order is not perturbed by the decoding. □

Table 3 summarizes the complexity of Rainbow Sort, independent of a specific implementation.

Input	$\Omega(n)$	$O(?)$
Processing	$\Theta(1)$	
Output	$\Omega(n)$	$O(?)$
Space	$\Omega(n)$	$O(?)$

Table 3: Complexity of Rainbow Sort (independent of a specific implementation). $O(?)$ indicates that a general upper bound cannot be given because it depends on the actual implementation.

The input step has the lower bound of $\Omega(n)$ because the for-loop is executed n times. The runtime of the processing step is in $\Theta(1)$, i.e., the actual sorting takes place in constant time

because the time interval until all rays have left the prism depends only on λ_{min}, which is a constant. This nice result arises due to the fact that the prism can "process" an arbitrary amount of rays in parallel. The lower bound for the output step is $\Omega(n)$ because the whenever-loop is iterated n times. The space complexity is in $\Omega(n)$ due to the output stack that stores all n elements. The upper bounds for the input and output steps and for the space complexity cannot be specified in general since they depend on the actual implementation. Table 4 contains some upper bounds according to the remarks on the implementation that are made in Section 3.

3 Implementation

At this stage, we cannot present a completely elaborated, working implementation of Rainbow Sort. Instead, we point out some essential difficulties that arise when Rainbow Sort is implemented; in particular, the effects of the *Heisenberg uncertainty principle* are discussed in Section 3.1. Furthermore, we make some proposals for a possible implementation. An exceptional property of Rainbow Sort is the fact that the processing step is by far the simplest one; the difficulties appear on the input and output side. Therefore, we concentrate on the input and output part in the Sections 3.2 and 3.3.

3.1 The Heisenberg Uncertainty Principle

According to the Heisenberg uncertainty principle [You04, p. 1498], the uncertainty ΔE of the energy depends on the time interval Δt during which the system remains in the given state. The relation is

$$\Delta E \cdot \Delta t \geq \frac{h}{2\pi}, \tag{4}$$

where h is Planck's constant. This can be restated in the following way: the more precise the measurement of the energy E, the more time t is needed. Furthermore, there is the following relation between the energy E of a photon and its wavelength λ [Hec02, p. 57]:

$$E = \frac{hc}{\lambda} \tag{5}$$

Let $E_1 = \frac{hc}{\lambda_1}, E_2 = \frac{hc}{\lambda_2}, \Delta E = E_2 - E_1$, and $\Delta\lambda = \lambda_1 - \lambda_2$. Then $\Delta E = \frac{hc}{\lambda_2} - \frac{hc}{\lambda_1} = \lambda_1 hc - \lambda_2 hc = \Delta\lambda hc$. Hence, $\Delta E \propto \Delta\lambda$ and, in connection with (4),

$$\Delta\lambda \propto 1/\Delta t. \tag{6}$$

Thus, if we want to determine the wavelength of a ray, we can do so by measuring the energy of the incoming photons and using (5). However, due to (6), we cannot determine the wavelength by a measurement with an arbitrary precision in constant time.

An optimal encoding function f has the following property:

$$\exists\Delta\lambda : \forall x \in \{0, 1, 2, \ldots, m-1\} : f(x+1) - f(x) = \Delta\lambda,$$

i.e., any two adjacent wavelengths have the same difference $\Delta\lambda$, in other words, the range $[\lambda_{min}, \lambda_{max}]$ is evenly subdivided into m intervals of size $\Delta\lambda$ each. Since $\lambda_{max} - \lambda_{min}$ is fixed, we have $m \propto 1/\Delta\lambda$. With reference to (6), this leads to

$$m \propto \Delta t. \tag{7}$$

Therefore, we can conclude that

$$\text{the runtime of an operation that relies on a measurement is in } \Omega(m). \tag{8}$$

3.2 Input

In order to realize the light source that is needed on the input side, we can use a laser that can be *tuned continuously* over a range of wavelengths $[\lambda_{min}, \lambda_{max}]$ [Hec02, p. 600]. The tuning can be performed electronically or thermally, i.e., by heating so that the temperature dependence of the system is exploited [KBE+00]. It is difficult to evaluate the time complexity of the tuning process in order to determine the time interval that is required to select the appropriate wavelength with a sufficient accuracy. If we need to measure the energy of the generated ray in order to tune it correctly, the runtime of the input step increases from $\Omega(n)$ to $\Theta(n+m)$ because of (8). This does not exclude that there is a possible realization that does not rely on a measurement so that a runtime of $\Theta(n)$ is feasible.

In principle, there are two imaginable setups in order to generate a ray whose spectrum consists of n wavelengths. First, we could use n tunable lasers and interlink the generated rays into one single ray. Second, we can think of one single tunable laser and some kind of *light storage* that supports operations as used in the algorithm in Figure 5, i.e., it has to be possible to add wavelengths to an existing ray in the light storage and, finally, to emit the ray. While the first proposal is probably realizable with present-day technology, the second one seems rather to be up in the air.

3.3 Output

The output is the most difficult part. In this section, some ideas are outlined that concern the implementation of the detector.

- The author's first idea was to use a very short pulse, ideally consisting of only one photon for each wavelength, and to measure the energy of the incoming photons in order to determine their wavelengths. The advantage would be that at any point in time only one photon would arrive at the detector so that the detector could not get confused by several rays that are incident simultaneously. Although it is feasible to create very short pulses (in the order of 100 attoseconds = 10^{-16} s) [Ser01], this idea has to be dismissed since such a short pulse cannot be used to measure the energy with a sufficient accuracy because of (4).

- Another approach could be the usage of longer pulses and of a detector that determines the smallest incoming wavelength by a measurement. However, this leads to a runtime complexity of $\Theta(n+m^2)$.

  Proof:
  Let us assume that the detector is not necessarily a straight line, but it is curved in such a way that the time interval between the arrival of any two adjacent wavelengths is the same Δt.

  Then, we have $T = m \cdot \Delta t$, where T is the time interval between the arrival of λ_{max} and λ_{min}. In connection with (7), we obtain $T \propto m \cdot m$. With reference to the lower bound of $\Omega(n)$ (cp. Table 3), this results in the runtime complexity of $\Theta(n+m^2)$. □

- The measurement of the energy can be avoided if the detector is subdivided into m cells where each cell y corresponds one-to-one with a fixed wavelength $g(y)$ from the range of the encoding function f. When a ray arrives at a cell y, the wavelength of this ray is known to be $g(y)$, without the need of doing a measurement. Hence, the runtime improves to $\Theta(n+m)$.

  Proof:
  The same argument that was used in the previous proof leads again to $T = m \cdot \Delta t$. But in this case, (7) does not apply as no measurement of the energy has to be performed. Hence, Δt need not be adapted when m is increased, but it is constant. Thus, $T \propto m$, which leads to a running time of $\Theta(n+m)$. □

  The space complexity of this implementation is in $\Theta(n+m)$ as well because in addition to the output stack of size n, we need the m cells of the detector.

Of course, this list is *not* complete. We cannot exclude that there is a better implementation that improves the upper bound of the running time and approaches the lower bound. For example, it could be worth considering the following idea:

- The measurement of the point in time when a ray arrives at the detector could be used in order to determine the represented number. Let us assume again that the time interval between the arrival of any two adjacent wavelengths is the same Δt. Furthermore, let t_{min} be the point in time when λ_{max} arrives. Then, we can conclude that a ray that arrives at time t represents the number $x = m - (t - t_{min})/\Delta t$. Hence, we "just" have to determine the points of time when the rays arrive and perform this simple calculation each time. In this case, (8) does *not* apply because we are not interested in the energy of the incoming photons, but we only measure the point in time when they arrive.

4 Conclusion

Table 4 combines the results from Section 2 and 3 concerning the complexity of Rainbow Sort. The upper bounds match with those of Counting Sort as mentioned in Section 1. The lower bounds of Rainbow Sort are equal to the trivial lower bounds for sorting. If we use an implementation that relies on measurements of the energy, the lower bound rises and we cannot do better than $\Theta(n+m)$ due to (8). Otherwise, if we can do without a measurement, the complexity of Rainbow Sort will be somewhere between $\Omega(n)$ and $O(n+m)$ depending on the best implementation that can be found.

Input	$\Omega(n)$	$O(n+m)$
Processing	$\Theta(1)$	
Output	$\Omega(n)$	$O(n+m)$
Space	$\Omega(n)$	$O(n+m)$

Table 4: Complexity of Rainbow Sort. The upper bounds base on the remarks on the implementation that are made in Section 3.

The optimal treatment of duplicates is an open question. A simple workaround is the adaption of the encoding function f so that each occurrence of the same number x is mapped to a different wavelength. This can be done by extending the domain of f from $\{0, 1, 2, \ldots, m\}$ to $[0, m+1]$, while the range stays unmodified. When a number x, whose index in the input array is i, is encoded, we take the value of $f_2(x, i) := f(x + i/n)$ instead of just $f(x)$. Therefore, since two duplicates have different indices, they are mapped to different wavelengths. However, depending on the actual implementation, this can have a negative impact on the running time because the distance between adjacent wavelengths can get smaller so that we might need more time in order to distinguish between the different wavelengths. For example, let us assume that the first number (index 0) is 1 and the last number (index $n-1$) is 0. As f is strictly monotonic increasing, the difference between $f_2(1, 0) = f(1)$ and $f_2(0, n-1) = f(\frac{n-1}{n})$ is smaller than the difference between $f(1)$ and $f(0)$.

Another open question is the preservation of the links between the keys and the records that contain the actual data. This is an important issue because practically in every application not only numbers have to be sorted, but keys that are linked to data records. For instance, in a database, whole records consisting of first name, surname and address are sorted by surname.

In order to preserve these links, we have to find a possibility to encode for each key x the address of the corresponding data record $a_x \in A$ into the ray that has the wavelength $f_2(x, i)$, where A denotes the address space.

- Again, this could be done by an appropriate adaption of the encoding function, which would take three parameters (the number x, the index i, and the address a_x) and would map such a triple one-to-one to a wavelength between λ_{min} and λ_{max}. However, again, this could impair the runtime.

- Another imaginable option is to *polarize* the light in order to store additional information.
- Alternatively, the duration of the laser pulses could be varied to encode the addresses of the data records. Let us assume that we have an implementation where the input part is realized by n lasers and the output part is constructed in such a way that the duration of the laser pulses has to be at least t_1 in order to allow precise results. Then, we can select a fixed $t_2 > t_1$ and use an encoding function $h : A \to [t_1, t_2]$ that maps each address a_x one-to-one to a duration $h(a_x)$. Thus, the laser that represents the number x generates a light pulse that has the wavelength $f_2(x, i)$ and the duration $h(a_x)$. The detector can measure the duration t of the light pulse and apply the decoding function h^{-1} in order to determine the address $h^{-1}(t)$ of the record that belongs to the key that is represented by the incoming light pulse.

The latter two approaches could also turn out to be useful with respect to the special treatment of duplicates. For example, the number of occurrences of each number x could be encoded using a function that is similar to h.

Acknowledgements

I would like to thank my friend Sebastian Will for his numerous useful comments from the physicist's point of view, which were encouraging and helped me to distinguish between the things that are already feasible with present-day technology, the things that could be realized in the future, and the things that are infeasible because of basic physical principles.

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A Simulator

A simulator for Rainbow Sort has been implemented in Java. The program, including the source code and a short note on the usage, is available online [Sch04]. The main purpose of the current version of the simulator is the demonstration of the basic idea, the setup, and the procedure of Rainbow Sort. It is not intended to simulate the physical concepts in every detail, but some simplifications have been made. The dispersion is implemented by a *linear* interpolation between a minimum and a maximum refraction index, which does not match with the reality (cp. Figure 3). Furthermore, the difference between the minimum and the maximum refraction index has been

selected to be quite large in comparison with the actual difference. Otherwise, it would be more difficult to follow the optical paths of the different rays.

The simulation is divided into discrete steps. During each step all rays are moved forward according to their current speed. If a ray intersects an edge of the prism, it is refracted according to Snell's Law (1) and its speed is adapted to the material. If a ray intersects the detector, it is decoded and added to the output. Since the time interval of each step is constant, while m is variable, problems can arise for large m when during one step several rays arrive "simultaneously" – at least they seem to arrive simultaneously. As a workaround, all rays that arrive at the same time step are cached, and after each step, the contents of the cache is sorted conventionally and added to the output.

In the simulation only visible light in the range of $[380\text{nm}, 780\text{nm}]$ is used for the obvious reason that invisible light would be a little bit boring for the human user. However, Rainbow Sort is not restricted to visible light, even though the name may suggest this. The optimal choice of λ_{min} and λ_{max} depends on the physical implementation, e.g., on the range of wavelengths that can be generated by tunable lasers, and not on the ability of the human eye.